Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2025

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Leaving Certificate 2025

Mathematics

Higher Level

Paper 1

Structure of the marking scheme – Paper 1

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled **A** divide candidate responses into two categories (correct and incorrect), scales labelled **B** divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
Number of categories	2	3	4	5
5-mark scales		0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10-mark scales			0, 4, 6, 10	0, 2, 4, 6, 10
15-mark scale			0, 4, 8, 15	0, 4, 7, 10, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

General notes on the marking – Paper 1

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded. The only marks that may be awarded for a question are those on the scales above, or *Full Credit -1*.

Instructions regarding penalties for omitted or incorrect units are given in the scheme for each question to which they apply. A penalty for rounding is applied once per unit of marking (i.e., if parts (a) and (b) are marked as a single unit, a penalty for rounding is only applied once for (a) and (b) combined).

In general, an answer without sufficient supporting work is awarded the lowest level of credit above *No Credit* (typically *Partial Credit* or *Low Partial Credit*, as appropriate).

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Steps
Where steps are listed in the Marking Notes, unless otherwise specified, it is to be taken that they can be independently correct / incorrect – that is, in a candidate's solution, step n can be considered correct even if previous step(s) have not been correctly presented, as long as the work done to arrive at step n from the previous step(s) has not been oversimplified. It is specifically noted where this does not hold. Note also that these steps may not need to be presented in the order specified in the Marking Notes.

Errors
Where a question is **not** marked using steps, if a candidate has a single error, they are generally awarded one level of credit below that which they would otherwise have been awarded. Similarly, where they have two errors, they are generally awarded two levels of credit below that which they would otherwise have been awarded. (If they present sufficient work for *Low Partial Credit*, they will be awarded this at a minimum, regardless of the number of errors.) For example, on a C-scale:

- *High Partial Credit*: One error, otherwise fully correct (or fully correct with a *)
- *Low Partial Credit*: Two errors, otherwise fully correct (or fully correct with a *)

Where a question **is** marked using steps, this does not apply. Instead, an error in a step means that the step has not been completed correctly; this does not affect the completion of other steps (unless it oversimplifies the work). So, if a candidate has multiple errors on a single step, they could still be awarded up to *High Partial Credit*, depending on the marking scheme.

The *
Where a candidate has a single * on their solution, this is ignored in the awarding of credit unless they would otherwise have *Full Credit*. Where a candidate has multiple *s, this is generally treated as an error.

Multiple answers
Where the solution requires substantial work, mark all separate attempts and award the marks for the best one, regardless of crossing out.

Where a solution requires selection from the question:

- If a candidate has crossed out answer(s), ignore the crossed-out answers
- If a candidate has multiple answers that are **not** crossed out, award the lowest mark associated with these answers (generally, this will be considered incorrect)

Square brackets
Where something is contained in square brackets in the model solution, it is **not** required for *Full Credit*.

Work of merit
Where the scheme indicates "work of merit", examples are given that exemplify the standard of work required to be considered work of merit in that particular question.

Palette of annotations available to examiners – Paper 1

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✓	Tick	Work of relevance	The work presented in the body of the script merits full credit
✗	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding / Unit / Arithmetic error / Misreading	
	Horizontal wavy	Error	
P	Partial Credit		The work presented in the body of the script merits *Partial Credit*
L	Low Partial Credit		The work presented in the body of the script merits *Low Partial Credit*
M	Mid Partial Credit		The work presented in the body of the script merits *Mid Partial Credit*
H	High Partial Credit		The work presented in the body of the script merits *High Partial Credit*
F*	F star		The work presented in the body of the script merits *Full Credit – 1*
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
	Vertical wavy	No work on this page / portion of this page	
O	Oversimplify	The candidate has oversimplified the work	
WOM	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
S	Stops early	The candidate has stopped early in this part	

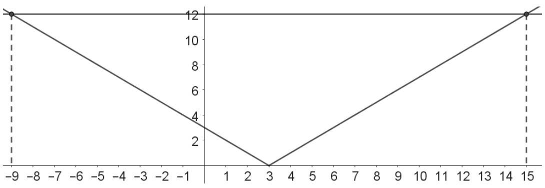
Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and and appear in the body of the work, then L should be placed in the right margin.

In the case of a **D scale** with the same annotations, M should be placed in the right margin.

Model Solutions & Marking Notes – Paper 1

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1:** $x-3 \leq 12$ and $x-3 \geq -12$ $x \leq 15$ and $x \geq -9$ $-9 \leq x \leq 15$ **Method 2:** $(x-3)^2 \leq 12^2$ $x^2-6x+9 \leq 144$ $x^2-6x-135 \leq 0$ Roots: $x=\frac{6 \pm \sqrt{(-6)^2-4(1)(-135)}}{2(1)}$ $x=15$ and $x=-9$ $-9 \leq x \leq 15$ **Method 3**  $-9 \leq x \leq 15$	**Scale 10D (0, 2, 4, 6, 10)** Accept correct answer without work Accept: $x \leq 15$ and $x \geq -9$ Accept: $x \leq 15 \cap x \geq -9$ *Low Partial Credit:* • Work of merit, for example, writes $x-3=-12$, or indicates squaring • Relevant effort at plotting $y=\|x-3\|$, for example, plots the line $y=x-3$ • Plots the line $y=12$ *Mid Partial Credit:* • Finds $x=15$ **or** $x=-9$ • Quadratic equation/inequality (0 on RHS) *High Partial Credit:* • Finds $x=15$ **and** $x=-9$ *Full Credit –1:* • Apply a * once if strict inequality / inequalities is used • *and* or ∩ is not included in Method 1

Q1	Model Solution – 30 Marks	Marking Notes
(b)	**Method 1** $(4x - 10\sqrt{x})(2x + 5\sqrt{x} - 7)$ $= 4x\left(2x + 5\sqrt{x} - 7\right)$ $\quad -10\sqrt{x}(2x + 5\sqrt{x} - 7)$ $= 8x^2 + 20x\sqrt{x} - 28x$ $-20x\sqrt{x} - 50x + 70\sqrt{x}$ $= 8x^2 - 78x + 70\sqrt{x}$ **Method 2** Let $y = \sqrt{x}$ $4y^2(2y^2 + 5y - 7) - 10y(2y^2 + 5y - 7)$ $= 8y^4 + 20y^3 - 28y^2 - 20y^3 - 50y^2 + 70y$ $= 8y^4 - 78y^2 + 70y$ $= 8x^2 - 78x + 70\sqrt{x}$	**Scale 10C (0, 4, 6, 10)** In the last two lines, there are 9 terms (6 in the penultimate line and 3 in the last line) to be checked. For *LPC* and *HPC*, excess terms can be ignored. For *FC*, there should be no excess terms. *Low Partial Credit:* • Work of merit, for example some correct work in distributing the terms • Correct answers without supporting work *High Partial Credit:* • 6 of the 9 terms in the last two lines correct (ignoring excess terms) • $8y^4 - 78y^2 + 70y$ (Method 2)

Q1	Model Solution – 30 Marks	Marking Notes
(c)	**Method 1** $$\begin{array}{r} 2x^2 - 9x + 10 \\ 2x + 3\overline{\smash{)}4x^3 - 12x^2 - 7x + 30} \\ \underline{4x^3 + 6x^2} \\ -18x^2 - 7x \\ \underline{-18x^2 - 27x} \\ 20x + 30 \\ \underline{20x + 30} \\ 0 \end{array}$$ $2x^2 - 9x + 10 = 0$ $(2x + 3)(2x - 5)(x - 2) = 0$ $x = -\frac{3}{2}$ and $x = \frac{5}{2}$ and $x = 2$ **Method 2** $$\begin{array}{c\|c\|c\|c\|} & 2x^2 & -9x & 10 \\ \hline 2x & 4x^3 & -18x^2 & 20x \\ \hline 3 & 6x^2 & -27x & 30 \\ \hline \end{array}$$ $2x^2 - 9x + 10 = 0$ $(2x + 3)(2x - 5)(x - 2) = 0$ $x = -\frac{3}{2}$ and $x = \frac{5}{2}$ and $x = 2$ **Method 3** $(2x + 3)(ax^2 + bx + c) = 4x^3 - 12x^2 - 7x + 30$ $(2x + 3)(2x^2 + bx + 10) = 4x^3 - 12x^2 - 7x + 30$ $20x + 3bx = -7x$ $(20 + 3b)x = -7x$ $20 + 3b = -7$ $b = -9$ $2x^2 - 9x + 10 = 0$ $(2x + 3)(2x - 5)(x - 2) = 0$ $x = -\frac{3}{2}$ and $x = \frac{5}{2}$ and $x = 2$	**Scale 10D (0, 2, 4, 6, 10)** Consider solution as consisting of 5 steps: **Step 1.** Sets up long division / array/product **Step 2.** Finds first term in quotient ($2x^2$ or 10) **Step 3.** Finds $2x^2 - 9x + 10$ **Step 4.** Factorises $2x^2 - 9x + 10$ / fully substituted quadratic formula **Step 5.** Finds 3 values of x *Low Partial Credit:* • 1 step correct • Finds 1 value of x • Correct answers without supporting work • Trials values *Mid Partial Credit:* • 2 (or 3) steps correct • Finds $x = -\frac{3}{2}$ **and** verifies either $x = \frac{5}{2}$ or $x = 2$ • Verifies both $x = \frac{5}{2}$ and $x = 2$ *High Partial Credit:* • 4 steps correct

<table>
<tr><th>Q2</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>$f'(x) = 2x + 4\cos 4x$</td><td>Scale 5C (0, 2, 3, 5)
Accept correct answer without work

Low Partial Credit:
• Some correct differentiation

High Partial Credit:
• One error in differentiating $\sin 4x$, otherwise correct, for example, mishandles the 4, or gets $-4\cos 4x$
• $\sin 4x$ differentiated correctly</td></tr>
<tr><td>(a)
(ii)</td><td>Slope of tangent at $x = 0$:
$$f'(0) = 2(0) + 4\cos(4(0))$$
$$= 4$$
Point at $x = 0$:
$$y = 6 + (0)^2 + \sin(4(0))$$
$$= 6$$
$\Rightarrow (0, 6)$ is the point of contact

Equation:
$y - 6 = 4(x - 0)$
Required form:
$4x - y + 6 = 0$
Or
$-4x + y - 6 = 0$</td><td>Scale 10D (0, 2, 4, 6, 10)
Consider solution as consisting of 4 steps:

Step 1. Finds slope of tangent when $x = 0$
Step 2. Finds y-value when $x = 0$
Step 3. Substitutes into equation of line formula
Step 4. Solution in required form

Steps 1 and 2 can be done in either order.
Both Steps 1 and 2 need to be done (though may have errors) to get credit in Step 3.
A value for the slope of the tangent is required to get credit in Step 4

Low Partial Credit:
• Work of merit, for example, states $f'(0)$ or $f(0)$, or some correct substitution into a formula for the equation of a line.
• Brings down the derivative from (a)(i)

Mid Partial Credit:
• 2 steps correct

High Partial Credit:
• 3 steps correct</td></tr>
</table>

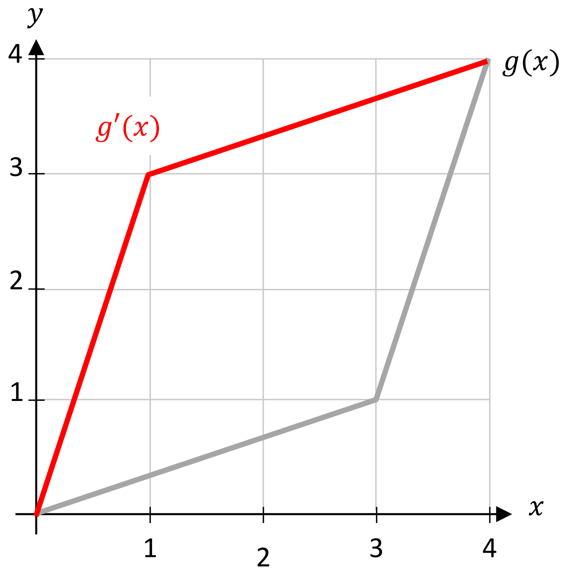
<table>
<tr><th>Q2</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(b)</td><td>

(i) $3 < x \leq 4$

(ii) $g(3) = 1$

$g(g(3)) = g(1) = \frac{1}{3}$

(iii)



</td><td>

Scale 15D (0, 4, 7, 10, 15)

Accept $3 \leq x \leq 4$ in part (i)

Accept $[3, 4]$ or $(3, 4)$ in part (i)

Accept $x \geq 3$ in part (i)

Low Partial Credit:

- Work of merit, for example, in **(i)**, indicates that $g'(x)$ is slope; range within $3 \leq x \leq 4$ in **(ii)**, finds $g(3)$ or relevant work on the diagram; in **(iii)**, draws $y = x$, or point $(1, 3)$ plotted

Mid Partial Credit:

- 1 part correct
- Work of merit in all 3 parts

High Partial Credit:

- 2 parts correct

Full Credit -1:

- Apply a * if answer in **(ii)** is within the interval $\left[\frac{1}{5}, \frac{2}{5}\right] \setminus \frac{1}{3}$
- Apply a * if work not shown on the diagram in part **(ii)**
- Apply a * if the graph of $g^{-1}(x)$ is not labelled in part **(iii)**

</td></tr>
</table>

<table>
<tr><th>Q3</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$f'(x) = 28(3x^5 - 4)^{27}(15x^4)$</td><td>Scale 5B (0, 2, 5)
Accept correct answer without work

Partial Credit:
• Work of merit, for example, correctly deals with the power of 28, or some correct differentiation of $3x^5$</td></tr>
<tr><td>(b)</td><td>Method 1
[By the quotient rule:]
$g'(x) = \frac{(2x-7)(0) - 3(2)}{(2x-7)^2}$
$= -\frac{6}{(2x-7)^2}$
$\neq 0$ [for all x in the domain of $g(x)$]
[So, the function has no turning points]
Method 2
[By the chain rule]
$g(x) = 3(2x-7)^{-1}$
$g'(x) = -3(2x-7)^{-2}(2)$
$= -6(2x-7)^{-2}$
< 0[for all x in the domain of $g(x)$]
[So, the function has no turning points]

Method 3
$g'(x) = -\frac{6}{(2x-7)^2}$
< 0[for all x in the domain of $g(x)$]

[So $g(x)$ is a decreasing function and therefore has no turning points]</td><td>Scale 10D (0, 2, 4, 6, 10)
Note: If quotient rule or chain rule not used award low partial credit at most

Consider solution as consisting of 3 steps:

Step 1. Unsimplified expression for $g'(x)$
Step 2. Simplified expression for $g'(x)$
Step 3. Explanation

Low Partial Credit:
• Work of merit, for example, some correct differentiation
• $g(x) = 3(2x-7)^{-1}$

Mid Partial Credit:
• 1 step correct

High Partial Credit:
• 2 steps correct</td></tr>
</table>

Q3	Model Solution – 30 Marks	Marking Notes
(c)	$\int_0^k e^{5x}dx = \left[\frac{1}{5}e^{5x}\right]_0^k$ $\left[\frac{1}{5}e^{5x}\right]_0^k = 9$ $\frac{1}{5}\left[e^{5(k)} - e^{5(0)}\right] = 9$ $e^{5k} - 1 = 45$ $e^{5k} = 46$ $5k = \ln 46$ $k = \frac{\ln 46}{5}$	**Scale 15D (0, 4, 7, 10, 15)** Consider solution as consisting of 4 steps: **Step 1.** Integrates e^{5x} **Step 2.** Substitutes in limits **Step 3.** Isolates e^{5k} **Step 4.** Finds k *Low Partial Credit:* • Work of merit, for example, integrates e^{5x} and gets he^{5x}, where $h \in \mathbb{R}$ *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct *Full Credit -1:* • k given in decimal form, rounded correctly

Q4	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1:** $\left(\frac{2+3i}{4-5i}\right)\left(\frac{4+5i}{4+5i}\right)=\frac{8+10i+12i+15i^2}{16+20i-20i-25i^2}$ $=\frac{-7+22i}{41}$ $=-\frac{7}{41}+\frac{22}{41}i$ **Method 2:** $\frac{2+3i}{4-5i}=a+bi$ $(4-5i)(a+bi)=2+3i$ $4a+4bi-5ai-5bi^2=2+3i$ Re: $4a+5b=2$ Im: $-5a+4b=3$ Re × 5: $20a+25b=10$ Im × 4: $\underline{-20a+16b=12}$ $41b=22 \quad \therefore b=\frac{22}{41}$ $4a+5\left(\frac{22}{41}\right)=2 \quad \therefore a=-\frac{7}{41}$ $\frac{2+3i}{4-5i}=-\frac{7}{41}+\frac{22}{41}i$ **Method 3** Let $\theta_1=\tan^{-1}\frac{3}{2}$ Let $\theta_2=\tan^{-1}-\frac{5}{4}$ $2+3i=\sqrt{13}(\cos(\theta_1)+i\sin(\theta_1))$ $4-5i=\sqrt{41}(\cos(\theta_2)+i\sin(\theta_2))$ $\frac{2+3i}{4-5i}=\frac{\sqrt{13}(\cos(\theta_1)+i\sin(\theta_1))}{\sqrt{41}(\cos(\theta_2)+i\sin(\theta_2))}$ $=\frac{\sqrt{13}}{\sqrt{41}}(\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2))$ $=-\frac{7}{41}+\frac{22}{41}i$	**Scale 5D (0, 2, 3, 4, 5)** Accept: $a=-\frac{7}{41}$, $b=\frac{22}{41}$ Consider solution as consisting of 4 steps: **Method 1:** **Step 1.** Indicates multiplication of top and bottom by conjugate of denominator **Step 2.** Expands top line **Step 3.** Expands bottom line **Step 4.** Writes in the form $a+bi$ **Method 2:** **Step 1.** $2+3i=4a+4bi-5ai-5bi^2$ **Step 2.** Sets Re = Re and Im = Im **Step 3.** Solves for 1 variable (a or b) **Step 4.** Solves for 2nd variable **Method 3:** **Step 1.** Writes $2+3i$ in polar form **Step 2.** Writes $4-5i$ in polar form **Step 3.** Evaluates $\frac{2+3i}{4-5i}$ in polar form **Step 4.** Writes in the form $a+bi$ Note: If the argument is approximated using decimals, then *High Partial Credit* at most *Low Partial Credit:* • Work of merit, for example, writes the conjugate of the denominator, or states $2+3i=(4-5i)(a+bi)$ *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct *Full Credit –1:* • $\frac{-7+22i}{41}$

Q4	Model Solution – 30 Marks	Marking Notes
(b)	$(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$ $\text{LHS} = \cos^2\theta + 2i\cos\theta\sin\theta + i^2\sin^2\theta$ $= \cos^2\theta + 2i\cos\theta\sin\theta - \sin^2\theta$ Equate Reals with Reals: $\cos 2\theta = \cos^2\theta - \sin^2\theta$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, $(\cos\theta + i\sin\theta)^2 = (\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$ *High Partial Credit:* • $\cos^2\theta + 2i\cos\theta\sin\theta + i^2\sin^2\theta$
(c)	$z^6 = -64i$ so $z = (-64i)^{\frac{1}{6}}$ **Reference angle:** $\theta = -90°$ or $270°$ or $\frac{3\pi}{2}$ or $-\frac{\pi}{2}$ **Modulus:** $r = 64$ **General polar form:** $z = \left[64\left(\cos\left(\frac{3\pi}{2} + 2n\pi\right) + i\sin\left(\frac{3\pi}{2} + 2n\pi\right)\right)\right]^{\frac{1}{6}}$ **De Moivre's Theorem:** $z = 64^{\frac{1}{6}}\left[\cos\left(\frac{\pi}{4} + \frac{n\pi}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{n\pi}{3}\right)\right]$ <u>**ANY TWO OF:**</u> $\boldsymbol{n = 0}$: $2\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) = \sqrt{2} + \sqrt{2}i$ $\boldsymbol{n = 1}$: $2\left(\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)\right)$ $= \frac{-\sqrt{6}+\sqrt{2}}{2} + \frac{\sqrt{6}+\sqrt{2}}{2}i$ $\boldsymbol{n = 2}$: $2\left(\cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)\right) =$ $= \frac{-\sqrt{6}-\sqrt{2}}{2} + \frac{\sqrt{6}-\sqrt{2}}{2}i$ $\boldsymbol{n = 3}$: $2\left(\cos\left(\frac{\pi}{4} + \pi\right) + i\sin\left(\frac{\pi}{4} + \pi\right)\right)$ $= -\sqrt{2} - \sqrt{2}i$ $\boldsymbol{n = 4}$: $2\left(\cos\left(\frac{\pi}{4} + \frac{4\pi}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right)\right) =$ $= \frac{\sqrt{6}-\sqrt{2}}{2} + \frac{-\sqrt{6}-\sqrt{2}}{2}i$ $\boldsymbol{n = 5}$: $2\left(\cos\left(\frac{\pi}{4} + \frac{5\pi}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{5\pi}{3}\right)\right) =$ $= \frac{\sqrt{6}+\sqrt{2}}{2} + \frac{-\sqrt{6}+\sqrt{2}}{2}i$	**Scale 15D (0, 4, 7, 10, 15)** Polar form must be used to achieve any credit. Accept correct polar form without work (i.e., finding $\boldsymbol{r}$ and $\boldsymbol{\theta}$). General polar form is **not** required to find the roots. As the function z^6 is even, award *FC* if one root is found, and the second root is minus 1 times the first. Consider solution as consisting of 4 steps: **Step 1.** Finds θ **Step 2.** Finds r **Step 3.** 1 root evaluated using De Moivre's Theorem **Step 4.** 2nd root found *Low Partial Credit:* • Work of merit, for example, plots $-64i$, or indicates $z = (-64i)^{\frac{1}{6}}$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • Roots found correctly, but one or both in polar form with argument simplified, or one or both in decimal form

Q5	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1** $5x^2 + 20x - 12$ $= 5\left[x^2 + 4x - \frac{12}{5}\right]$ $= 5\left[(x^2 + 4x + 2^2) - \frac{12}{5} - 2^2\right]$ $= 5\left[(x + 2)^2 - \frac{32}{5}\right]$ $\therefore g(x) = 5(x+2)^2 - 32$ **Method 2** $5x^2 + 20x - 12 = a(x+h)^2 + k$ $= a(x^2 + 2xh + h^2) + k$ $= ax^2 + 2axh + ah^2 + k$ Equating coefficients of like terms: $a = 5$ $2ah = 20$ $2(5)(h) = 20$ $h = 10$ $ah^2 + k = -12$ $5(2)^2 + k = -12$ $k = -32$ $\therefore g(x) = 5(x+2)^2 - 32$ **Method 3** $f'(x) = 10x + 20$ $f'(x) = 0$ $10x + 20 = 0$ $x = -2$ [axis of symmetry] $f(-2) = -32$ $\therefore g(x) = 5(x+2)^2 - 32$	**Scale 10D (0, 2, 4, 6, 10)** Accept correct answer without work *Low Partial Credit:* • Work of merit, for example, factors out 5 from two terms, or squares out $(x + h)^2$ • Some correct differentiation • Finds a *Mid Partial Credit:* • Find h or k • Finds the turning point *High Partial Credit:* • Finds or identifies any two of a, h or k • Finds the turning point and identifies a

Q5	Model Solution – 30 Marks	Marking Notes
(b)	**Method 1** $\ln[(e^3p)^5] = 5\ln(e^3p)$ $= 5(\ln e^3 + \ln p)$ $= 5(3 + \ln p)$ $= 15 + 5\ln p$ **Method 2** $\ln[(e^3p)^5] = \ln e^{15}p^5$ $= \ln e^{15} + \ln p^5$ $= 15\ln e + 5\ln p$ $= 15 + 5\ln p$	**Scale 10C (0, 4, 6, 10)** **Method 1** Consider solution as consisting of 3 steps: **Step 1.** Deals with power of 5 **Step 2.** Splits up e^3p **Step 3.** Finishes **Method 2** Consider solution as consisting of 3 steps: **Step 1.** Splits up $e^{15}p^5$ **Step 2.** Deals with powers **Step 3.** Finishes *Low Partial Credit:* • Work of merit, for example, $\ln(e^3p)^5 = \ln(e^{15}p^5)$ • 1 step correct *High Partial Credit:* • 2 steps correct
(c)	**Method 1** One solution on the $y-$axis: $(0, y)$ $2(0) - y = 7$ $y = -7$ $\therefore$ $(0, -7)$ is a solution. Substitute $(0, -7)$ in the non-linear equation: $(0)^2 + (-7) + 2(-7)^2 = n$ $n = 91$ **Method 2** $y = 2x - 7$ $x^2 + (2x - 7) + 2(2x - 7)^2 = n$ $9x^2 - 54x + 91 = n$ At $x = 0$ $9(0)^2 - 54(0) + 91 = n$ $n = 91$	**Scale 10D (0, 2, 4, 6, 10)** **Method 1** Consider solution as consisting of 4 steps: **Step 1.** States $x = 0$ **Step 2.** Finds y when $x = 0$ **Step 3.** Substitutes $(0, -7)$ into the curve **Step 4.** Finds n **Method 2** Consider solution as consisting of 4 steps: **Step 1.** Writes y in terms of x **Step 2.** Substitutes the expression for y into the curve **Step 3.** Substitutes $x = 0$ **Step 4.** Finds n *Low Partial Credit:* • 1 step correct *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct

Q6	Model Solution – 30 Marks	Marking Notes
(a)	$\binom{7}{0}(2p)^7+\binom{7}{1}(2p)^6(3)+\binom{7}{2}(2p)^5(3)^2$ $=128p^7+1344p^6+6048p^5$	**Scale 10D (0, 2, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, one binomial coefficient correct, or writes $(2p)^7$, • Writes $(2p+3)(2p+3)$ or similar *Mid Partial Credit:* • 1 of the first three relevant terms found correctly • Part of 2 relevant terms correct (binomial coefficient, $(2p)^k$, or 3^k) *High Partial Credit:* • $(2p+3)^7$ expanded correctly but relevant three terms not identified • First 3 terms in ascending powers of p simplified • 1 error, otherwise correct (note that the error could be, for example, consistent mishandling of $(2p)$, or powers adding to the wrong number in each term)

Q6	**Model Solution – 30 Marks**	**Marking Notes**
(b)(i)	**Method 1** One solution $\Rightarrow b^2 - 4ac = 0$ $(-4r)^2 - 4(6m)(54m) = 0$ $16r^2 - 1296m^2 = 0$ $r^2 - 81m^2 = 0$ $(r - 9m)(r + 9m) = 0$ $r = 9m\ [\,r, m > 0]$ **Method 2** One solution $\Rightarrow$ 2 equal roots $\{\alpha, \alpha\}$ Product of roots: $\alpha^2 = \dfrac{54m}{6m}$ $= 9$ $\alpha = \pm 3$ Sum of roots: $2\alpha = \dfrac{4r}{6m}$ $[r, m > 0,$ $\Rightarrow \alpha > 0]$ $\alpha = 3$ $2(3) = \dfrac{4r}{6m}$ $4r = 36m$ $r = 9m$	**Scale 10D (0, 2, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, $b^2 - 4ac = 0$ or some correct substitution in the quadratic formula • Product of the roots $= \frac{54m}{6m}$ • Substitutes $9m$ for r into the equation and solves to show 1 root only *Mid Partial Credit:* • $(-4r)^2 - 4(6m)(54m) = 0$ • Identifies roots as ± 3 *High Partial Credit:* • $16r^2 - 1296m^2 = 0$ • $\frac{4r}{6m} = 6$

Q6	Model Solution – 30 Marks	Marking Notes
(b)(ii)	**Method 1** $x = \dfrac{-b \pm \sqrt{0}}{2a}$ $= \dfrac{4(9m)}{2(6m)}$ $= 3$ **Method 2** $6mx^2 - 36mx + 54m = 0$ $x^2 - 6x + 9 = 0$ $(x-3)^2 = 0$ $x = 3$ **Method 3** One solution, $\alpha \Rightarrow h'(\alpha) = 0$ $h'(\alpha) = 12m\alpha - 4r$ $12m\alpha - 4r = 0$ $12m\alpha - 36m = 0$ $\alpha - 3 = 0$ $\alpha = 3$ **Method 4** Product of roots: $\alpha^2 = \dfrac{54m}{6m}$ $= 9$ $\alpha = \pm 3$ Sum of roots: $2\alpha = \dfrac{4r}{6m}$ $[r, m > 0,$ $\Rightarrow \alpha > 0]$ $\alpha = 3$	**Scale 10C (0, 4, 6, 10)** Note: The supporting work for (b)(ii) may appear in the box for (b)(i) *Low Partial Credit:* • Work of merit, for example, $\dfrac{-b}{2a}$ or states $h'(x) = 0$ *High Partial Credit:* • $\frac{4(9m)}{2(6m)}$ (Method 1) • $(x-3)^2 = 0$ or equivalent (Method 2) • $12mx - 36m = 0$ (Method 3) • $\frac{54m}{6m}$ (Method 4)

Q7	Model Solution – 50 Marks	Marking Notes
(a) **(i)**	$A(2) = 37 \cdot 8 - 15 \cdot 8$ $= 22$ $A(3) = 66 - 37 \cdot 8$ $= 28 \cdot 2$	**Scale 5B (0, 2, 5)** Note: To find $A(3)$, it cannot be assumed that $A(n)$ is arithmetic *Partial Credit:* • Shows $A(2) = 22$ or $A(3)$ correct. • 22 shown on the table and no further work *Full Credit -1:* • Values for $A(2)$ and $A(3)$ not shown on the table
(a)(ii) **(iii)**	**(ii)** $15.8 \xrightarrow{6.2} 22 \xrightarrow{6.2} 28.2$ $d = 6{\cdot}2, \quad a = 15{\cdot}8$ $A(n) = 15{\cdot}8 + (n-1)(6{\cdot}2)$ $[A(n) = 6{\cdot}2n + 9{\cdot}6]$ **(iii)** $A(100) = 6 \cdot 2(100) + 9 \cdot 6$ $= 629 \cdot 6 \left(or\ \frac{3148}{5}\right)$	**Scale 10D (0, 2, 4, 6, 10)** For parts **(ii)** and **(iii)** allow full credit correct answers presented in either grid *Low Partial Credit:* • Work of merit in one part, for example, in **(ii)**, indicates common difference or identifies a; in **(iii)**, finds $A(4)$ (in **(iii)**, must involve moving beyond $A(3)$ in order to qualify as work of merit) *Mid Partial Credit:* • 1 part correct • Work of merit in both parts *High Partial Credit:* • 1 part correct **and** work of merit in the other part.

Q7	Model Solution – 50 Marks	Marking Notes
(a)(iv)	**Method 1** $S(n)=\frac{n}{2}(2(15\cdot 8)+(n-1)6\cdot 2)$ $=n(15\cdot 8)+n(3\cdot 1n)-n(3\cdot 1)$ $=3\cdot 1n^2+12\cdot 7n$ **Method 2** $S_n-S_{n-1}=T_n$ $3.1n^2+12.7n-\big(3.1(n-1)^2+12.7(n-1)\big)$ $=3.1n^2+12.7n-3.1n^2-6.5n+9.6$ $=6.2n+9.6$ $=T_n$ **Method 3** The cumulative amount of silk required increases in a quadratic pattern $S(1)$ $S(2)$ $S(3)$ 15.8 37.8 66 22 28.2 6.2 $S_n=an^2+bn+c$ $a=\frac{6.2}{2}=3.1$ $S_1=3.1(1)^2+b(1)+c=15.8$ $b+c=12.7$ $S_2=3.1(2)^2+b(2)+c=37.8$ $2b+c=25.4$ $b=12.7$ and $c=0$ $S_n=3\cdot 1n^2+12\cdot 7n$ **Method 4** In an arithmetic series the partial sums form a quadratic pattern. $15\cdot 8, 37\cdot 8$ and 66 form a quadratic pattern Verify $S(1)=15\cdot 8, S(2)=37\cdot 8, S(3)=66$ $S(1)=3\cdot 1(1)^2+12\cdot 7(1)=15\cdot 8$ $S(2)=3\cdot 1(2)^2+12\cdot 7(2)=37\cdot 8$ $S(3)=3\cdot 1(3)^2+12\cdot 7(3)=66$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Formula for the sum or an arithmetic series, with some relevant substitution • States $T_n=S_n-S_{n-1}$ • $A(1)+A(2)$ found *High Partial Credit:* • Fully substituted formula • S_n-S_{n-1} in terms of n • Two of a, b or c found • Verifies any two of $S(1)$, $S(2)$ or $S(3)$

Q7	Model Solution – 50 Marks	Marking Notes
(a)(v)	10 m $= 1000$ cm $3{\cdot}1n^2 + 12{\cdot}7n = 1000$ $3{\cdot}1n^2 + 12{\cdot}7n - 1000 = 0$ $n = \dfrac{-12{\cdot}7 \pm \sqrt{(12{\cdot}7)^2 - 4(3{\cdot}1)(-1000)}}{2(3{\cdot}1)}$ $n = 16{\cdot}02\ldots$ [as $n > 0$] $k = 17$ [as $k \in \mathbb{N}$]	**Scale 5D (0, 2, 3, 4, 5)** Consider solution as consisting of 4 steps: **Step 1**. Sets up quadratic equation **Step 2**. Fully substitutes quadratic formula **Step 3**. Solves the equation **Step 4**. Finds the value of k Note that incorrect rounding is effectively treated as an error instead of as a *, as it means Step 4 is not completed *Low Partial Credit:* • Work of merit, for example, states $3{\cdot}1n^2 + 12{\cdot}7n = 10$, or converts m to cm, or some correct substitution into quadratic formula • Trials $n = 16$ or $n = 17$ *Mid Partial Credit:* • 2 steps correct • Writes $3.1n^2 + 12.7n - 1000 = 0$ and concludes $k = 17$ *High Partial Credit:* • 3 steps correct • Mishandles or omits unit conversion, but otherwise correct.
(b)(i) **(ii)**	**(i)** $r = \dfrac{0{\cdot}53}{0{\cdot}5}$ $= 1.06 \left(or \dfrac{53}{50}\right)$ $O_3 = 1{\cdot}06(0{\cdot}53)$ $= 0{\cdot}5618 \left(or \dfrac{2890}{5000}\right)$ **(ii)** $S_n = \dfrac{a(r^n - 1)}{r - 1}$ $= \dfrac{0{\cdot}5(1{\cdot}06^n - 1)}{1{\cdot}06 - 1}$ $= \dfrac{25(1{\cdot}06^n - 1)}{3}$ or $\dfrac{-25(1 - 1{\cdot}06^n)}{3}$	**Scale 10D (0, 2, 4, 6, 10)** Accept $\frac{0{\cdot}5(1{\cdot}06^n - 1)}{1{\cdot}06 - 1}$ for full credit in Part (ii) *Low Partial Credit:* • Work of merit in some part, for example, in **(i)**, $\frac{0{\cdot}53}{0{\cdot}5}$, or in **(ii)**, some correct substitution in S_n formula. *Mid Partial Credit:* • 1 part correct • Work of merit in both parts *High Partial Credit* • 1 part correct **and** work of merit in the other part.

Q7	Model Solution – 50 Marks	Marking Notes
(b)(iii)	**Method 1** [$18k$ orbitals in the first k laps, so:] $S_{18k} = \frac{0{\cdot}5\left(1{\cdot}06^{18k} - 1\right)}{1{\cdot}06 - 1}$ $= \frac{25\left(1{\cdot}06^{18k} - 1\right)}{3}$ or $\frac{-25\left(1 - 1{\cdot}06^{18k}\right)}{3}$ **Method 2** Each lap forms a geometric sequence. $a = S_{18}$ and $r = \frac{S_{36} - S_{18}}{S_{18}}$ $S_{36} = \frac{25}{3}(1.06^{36} - 1)$ $S_{18} = \frac{25}{3}(1.06^{18} - 1)$ $r = \frac{(1.06^{36} - 1) - (1.06^{18} - 1)}{1.06^{18} - 1}$ $= \frac{1.06^{36} - 1.06^{18}}{1.06^{18} - 1}$ $= \frac{1.06^{18}(1.06^{18} - 1)}{1.06^{18} - 1}$ $= 1.06^{18}$ $S_k = \frac{\frac{25}{3}(1.06^{18} - 1)\left((1.06^{18})^k - 1\right)}{1.06^{18} - 1}$ $= \frac{25}{3}\left(1.06^{18k} - 1\right)$	**Scale 15C (0, 4, 8, 15)** Note: In Method 2 award High Partial Credit at most if S_{18} or S_{36} are approximated as decimals *Low Partial Credit:* • Some relevant substitution into S_n formula • kS_{18} *High Partial Credit:* • $S_{18k} = \frac{0{\cdot}5\left(1{\cdot}06^{18k} - 1\right)}{1{\cdot}06 - 1}$ • S_k not simplified (Method 2)

Q8	Model Solution – 50 Marks	Marking Notes
(a)	**Method 1** $870 \times 0{\cdot}85 \times 0{\cdot}9 = [€]665{\cdot}55$ **Method 2** $870 \times 0{\cdot}15 = 130.5$ $870 - 130.5 = 739.5$ $739.5 \times 0{\cdot}1 = 73.95$ $739.5 - 73.95 = [€]665{\cdot}55$	**Scale 5C (0, 2, 3, 5)** Accept correct answer without supporting work. *Low Partial Credit:* • Finds a relevant percentage of 870 (15% *or* 10%) *High Partial Credit:* • $870 \times 0{\cdot}85 \times 0{\cdot}9$ indicated • Finds 85% or 90% of 870
(b)	**Method 1** *Cost according to Jacob:* $\frac{95}{1.183} = €80.30\ldots$ *Actual cost:* $80{\cdot}30 + 1{\cdot}02 = €81{\cdot}32\ldots$ *Actual rate:* $d = \frac{95}{81 \cdot 32}$ $= 1{\cdot}1682\ldots$ (or $1{\cdot}1681\ldots$ if an actual cost with more decimal places is used) $d = 1{\cdot}168\ [3\mathrm{D.P.}]$ **Method 2** $\frac{95}{d} - \frac{95}{1{\cdot}183} = 1{\cdot}02$ $95(1{\cdot}183) - 95d = 1{\cdot}02(1{\cdot}183)d$ $112{\cdot}385 = 1{\cdot}20666\,d + 95\,d$ $d = \frac{112{\cdot}385}{96{\cdot}20666}$ $= 1{\cdot}1681\ldots$ $d = 1{\cdot}168[3\ \mathrm{D.P.}]$	**Scale 10D (0, 2, 4, 6, 10)** Consider solution as consisting of 3 steps: **Method 1** **Step 1**. Finds cost that Jacob thinks (in €) **Step 2**. Finds actual cost (in €) **Step 3**. Finds actual rate **Method 2** **Step 1**. Sets up equation in d **Step 2**. Simplifies to an equation without denominators **Step 3**. Solves for d *Low Partial Credit:* • Work of merit, for example, writes $\frac{95}{d}$ *Mid Partial Credit:* • 1 step correct *High Partial Credit:* • 2 steps correct *Full Credit -1:* • Apply a * for no rounding or incorrect rounding

<table>
<tr><th>Q8</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(c) (d)</td><td>

(c)

$$\text{Time} = \frac{2}{6} + \frac{8}{12}$$

$$= 1 \text{ hour}$$

(d)

$|SF|^2 = 8^2 + 2^2$

$|SF| = \sqrt{68}$

$$\text{Time} = \frac{2\sqrt{17}}{6}$$

$$= 1{\cdot}37 \ldots \text{hours}$$

$$= 82 \text{ minutes [nearest minute]}$$

$$[= 1 \text{ hr } 22 \text{ mins }]$$

</td><td>

Scale 15D (0, 4, 7, 10, 15)

Low Partial Credit:

- Work of merit, for example, in (c), one correct time found;
or, in (d), some correct substitution into Pythagoras’ Theorem

Mid Partial Credit:

- 1 part correct
- Work of merit in both parts

High Partial Credit:

- (d) correct and work of merit in (c)
- (c) correct and answer to (d) given in hours only

Full Credit –1

- Incorrect rounding, otherwise correct
- Incorrect or no units in (c)

</td></tr>
<tr><td>(e)(i)</td><td>

Method 1

$|SB| = \sqrt{x^2 + 4} \quad |BF| = 8 - x$

$$T = \frac{\sqrt{x^2 + 4}}{6} + \frac{8 - x}{12}$$

Method 2

$|SB| = \sqrt{x^2 + 4} \quad |BF| = 8 - x$

$\text{Time}_{|SB|} = 10\sqrt{x^2 + 4}$ [mins]

$\text{Time}_{|BF|} = 5(8 - x)$ [mins]

$\text{Total} = 10\sqrt{x^2 + 4} + 5(8 - x)$ [mins]

$$= \frac{10\sqrt{x^2 + 4} + 5(8 - x)}{60} \text{ [hours]}$$

</td><td>

Scale 5D (0, 2, 3, 4, 5)

Consider solution as consisting of 4 steps:

Step 1. Finds $|SB|$
Step 2. Finds time to travel $|SB|$
Step 3. Finds $|BF|$
Step 4. Finds total time taken

Low Partial Credit:

- Work of merit, for example, some correct substitution into Pythagoras’ Theorem,
or finds time to run x km,
or finds $8 - x$

Mid Partial Credit:

- 2 steps correct

High Partial Credit:

- 3 steps correct

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</table>

<table>
<tr><th>Q8</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(e)(ii)</td><td>

Method 1

$$\frac{x}{6\sqrt{x^2+4}} - \frac{1}{12} = 0$$

$$12x - 6\sqrt{x^2+4} = 0$$

$$2x = \sqrt{x^2+4}$$

$$4x^2 = x^2 + 4$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \frac{2}{\sqrt{3}}$$

$$= 1.1547\ldots$$

$$= 1.155\ [\text{km}][3\ \text{D.P.}]$$

Method 2

$$\frac{x}{6\sqrt{x^2+4}} = \frac{1}{12}$$

$$\frac{x^2}{36(x^2+4)} = \frac{1}{144}$$

$$144x^2 = 36x^2 + 144$$

$$108x^2 = 144$$

$$x^2 = \frac{144}{108}$$

$$= \frac{4}{3}$$

$$x = \frac{2}{\sqrt{3}}$$

$$= 1{\cdot}1547\ldots$$

$$= 1.155\ [\text{km}][3\ \text{D.P.}]$$

</td><td>

Scale 5D (0, 2, 3, 4, 5)

Consider solution as consisting of 4 steps:

Step 1. Gets rid of fractions
Step 2. Squares both sides
Step 3. Finds the correct quadratic equation with one term in x^2, for example, $3x^2 = 4$ or $108x^2 = 144$
Step 4. Finds x

Low Partial Credit:

- Work of merit, for example, some correct transposition

Mid Partial Credit:

- 2 steps correct

High Partial Credit:

- 3 steps correct

Full Credit -1:

- Incorrect or no rounding
- $\frac{2}{\sqrt{3}}$ or equivalent

</td></tr>
</table>

<table>
<tr><th>Q8</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(e)(iii)</td><td>Answer: D

Justification:

One turning point ⇒ C or D.

$T(0) = 1$ hour

$T(8) = 1$ hour 22 mins

$T(0) < T(8)$
$\Rightarrow D$</td><td>Scale 10C (0, 4, 6, 10)

Low Partial Credit:
• Correct answer
• Work of merit in justification, for example, states “one turning point” or similar

High Partial Credit:
• States “one turning point”, and states C or D
• States “$T(0) < T(8)$”, and states B or D
• Sufficient justification to support the correct answer, but doesn’t give correct answer</td></tr>
</table>

Q9	Model Solution – 50 Marks	Marking Notes
(a)(i)	**Method 1** $F(60) = 0{\cdot}05(60)^2 - 8{\cdot}5(60) + 800$ $= 470$ $F(110) = 0{\cdot}05(110)^2 - 8{\cdot}5(110) + 800$ $= 470$ $[= F(60)]$ **Method 2** $F'(c) = 0{\cdot}1c - 8{\cdot}5$ $0{\cdot}1c - 8{\cdot}5 = 0$ at local min $c = 85$ [axis of symmetry] $\dfrac{60 + 110}{2} = 85$ $\Rightarrow F(60) = F(110)$[by symmetry]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, substitutes in 60 or 110 for c, or finds $F'(c)$ *High Partial Credit:* • Evaluates $F(60)$ or $F(110)$ • Finds $c = 85$ when $F'(c) = 0$

<table>
<tr><th>Q9</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)(ii)
(iii)</td><td>(ii)
$\frac{dF}{dc} = 0{\cdot}1c - 8{\cdot}5$

(iii)
At $t = 7$:
$c = 78 + 9\ln(7^2)$
$\quad = 113{\cdot}02\ldots$
$\frac{dF}{dc} = (0{\cdot}1c - 8{\cdot}5)$
$\quad = (0{\cdot}1(113{\cdot}02\ldots) - 8{\cdot}5)$
$\quad = 2{\cdot}80\ldots$
$\quad = 2{\cdot}8\ [1\text{ D.P.}]$</td><td>Scale 10D (0, 2, 4, 6, 10)
Consider solution as consisting of 4 steps:
Step 1. (ii) correct
Step 2. In (iii), subs $t = 7$ into c
Step 3. In (iii), subs c (with $t = 7$) into $\frac{dF}{dc}$
Step 4. In (iii), evaluates $\frac{dF}{dc}$

If c is evaluated at $t = 7$ and this value is subbed into $\frac{dF}{dc}$, and the resulting expression is evaluated, then consider all the evaluating as comprising Step 4. So, if there are errors in evaluating both c and $\frac{dF}{dc}$ these are both treated as errors in Step 4, and up to HPC can still be awarded for 3 steps correct.

Low Partial Credit:
• Work of merit, for example, in (ii), some correct differentiation; in (iii), 7^2 evaluated

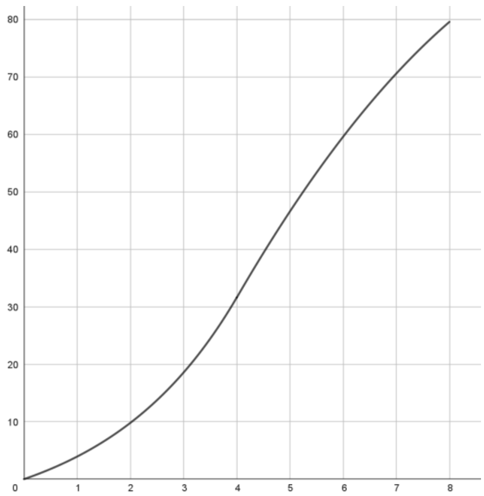
Mid Partial Credit:
• 2 steps correct

High Partial Credit:
• 3 steps correct

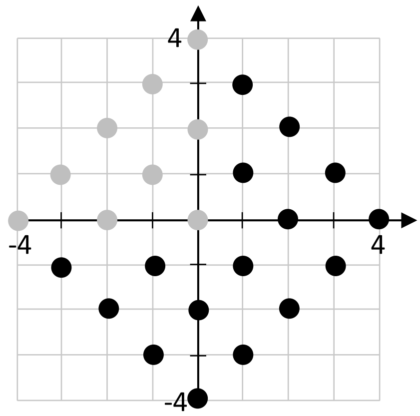
Full Credit –1:
• Apply a * for Incorrect rounding</td></tr>
<tr><td>(a)(iv)</td><td>Method 1
$\frac{dc}{dt} = 9\left(\frac{1}{t^2}\right)(2t)$
$\quad = \frac{18t}{t^2}$
$\quad = \frac{18}{t}$

Method 2
$c = 78 + 18\ln t$
$\frac{dc}{dt} = \frac{18}{t}$</td><td>Scale 5B (0, 2, 5)
Partial Credit:
• Some correct differentiation, for example, $\frac{1}{t^2}$
• $c = 78 + 18\ln t$</td></tr>
</table>

Q9	Model Solution – 50 Marks	Marking Notes
(a)(v)	$\frac{dF}{dt} = \frac{dF}{dc} \times \frac{dc}{dt}$ $= (2{\cdot}8) \times \left(\frac{18}{t}\right)$ $= (2{\cdot}8) \times \left(\frac{18}{7}\right)$ $= 7{\cdot}2$ or $\frac{36}{5}$ [(litres/10000 km) / minute]	**Scale 10D (0, 2, 4, 6, 10)** Accept correct answer without unit Consider solution as consisting of 4 steps: **Step 1**. $\frac{dF}{dt} = \frac{dF}{dc} \times \frac{dc}{dt}$ **Step 2**. Fills in value for $\frac{dF}{dc}$ **and** expression for $\frac{dc}{dt}$ **Step 3**. Fills in $t = 7$ in $\frac{dc}{dt}$ **Step 4**. Evaluates $\frac{dF}{dt}$ Steps can happen in different orders. Step 1 does not need to be explicitly stated. As in **(a)(iii)**, all evaluating is considered to be part of Step 4, and all errors in evaluating are considered to be restricted to Step 4. *Low Partial Credit:* • 1 step correct *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct

Q9	Model Solution – 50 Marks	Marking Notes
(b)(i) **(ii)**	**(i)** *Four missing values, in order, to 1 D.P. where appropriate (first two are* $3{\cdot}93\ldots$ *and* $18{\cdot}56\ldots$*):* $3{\cdot}9,\quad 18{\cdot}6,\quad 46{\cdot}6,\quad 59{\cdot}6$ **(ii)** 	**Scale 10D (0, 2, 4, 6, 10)** Consider solution as consisting of 14 parts: 4 parts: the 4 values in the table in **(i)** 9 parts: 9 points plotted from the values in the table 1 part: points joined appropriately *Low Partial Credit* • 2 parts correct *Mid Partial Credit* • 7 parts correct *High Partial Credit* • 12 parts correct *Full Credit –1* • Apply a * once for incorrect rounding of $v(1)$ and/or $v(3)$ • Fully correct apart from 1 part (excluding rounding error)

Q9	Model Solution – 50 Marks	Marking Notes
(b)(iii)	$\frac{1}{8-4}\left[\int_4^8(-t^2+24t-48{\cdot}4)dt\right]$ $= \quad \frac{1}{4}\left[\left(-\frac{t^3}{3}+12t^2-48{\cdot}4t\right)\right]_{t=4}^{t=8}$ $= \quad \frac{1}{4}\times\left[\left(-\frac{8^3}{3}+12(8^2)-48{\cdot}4(8)\right)-\left(-\frac{4^3}{3}+12(4^2)-48{\cdot}4(4)\right)\right]$ $= \quad \frac{1}{4}\times\left[\frac{3152}{15}-\left(-\frac{344}{15}\right)\right]$ $= \quad 58{\cdot}26\ldots$ $= 58{\cdot}3$ [km/hour] [1 D.P.]	**Scale 10D (0, 2, 4, 6, 10)** Note: Indication of integration is required to be awarded any credit Some correct integration needed to go beyond Low Partial Credit Accept correct answer without unit. Consider solution as consisting of 4 steps: **Step 1.** $\frac{1}{8-4}\left[\int_4^8 v(t)dt\right]$ **Step 2.** Integrates correctly **Step 3.** Subs in limits **Step 4.** Evaluates correctly If $\frac{1}{4}$ is omitted, treat Step 1 as not fully correct, but all other steps can be accepted as correct. *Low Partial Credit* • Work of merit, for example, integration indicated *Mid Partial* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit -1* • Apply a * for incorrect rounding • $4 < lower\ limit \leq 5$, otherwise correct

Q10	Model Solution – 40 Marks	Marking Notes
(a)		**Scale 5C (0, 2, 3, 5)** Note: If there are more than three excess points, award low partial credit at most *Low Partial Credit:* • 5 correct points (ignoring excess points) *High Partial Credit:* • 12 correct points (at most 3 excess points) *Full Credit –1:* • 1 point missing, or 1 incorrect points plotted, otherwise fully correct
(b) **(c)**	**(b)** $(0, 2000), (2000,0), (0, -2000), (-2000,0)$ **(c)** $(1, 1)$ in $P2$ $(2, 2)$ in $P4$ $(3, 3)$ in $P6$ $(4, 4)$ in $P8$ $\therefore n = 8$	**Scale 10D (0, 2, 4, 6, 10)** Accept correct answer without work *Low Partial Credit:* • Work of merit, for example, in **(b)**, one point given with 2000 as one of the co-ordinates; in **(c)**, draws some of pattern for a value of n where $n > 4$ • Shows $(4, 4)$ on any one of the diagrams *Mid Partial Credit:* • 1 part correct • Work of merit in both parts *High Partial Credit:* • 1 part correct **and** work of merit in the other part
(d) **(i)**	$n = \frac{t-1}{2}$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, $t - 1 = 2n$

Q10	Model Solution – 40 Marks	Marking Notes
(d) (ii)	$Q(n) = \frac{(n+1)^2}{(2n+1)^2}$ $= \frac{\left(\frac{t-1}{2}+1\right)^2}{\left(2\left(\frac{t-1}{2}\right)+1\right)^2}$ $= \frac{\left(\frac{t-1+2}{2}\right)^2}{(t-1+1)^2}$ $= \frac{\left(\frac{(t+1)^2}{2^2}\right)}{t^2}$ $= \frac{t^2+2t+1}{4t^2}$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Substitutes in $n = \frac{t-1}{2}$ • Substitutes $t = 2n + 1$ into $\frac{t^2+2t+1}{4t^2}$ *High Partial Credit:* • $\frac{\left(\frac{(t+1)^2}{2^2}\right)}{t^2}$ or similar • Substitutes $n = \frac{t-1}{2}$ and completes squaring • $\frac{\left(\frac{t-1}{2}\right)^2+2\left(\frac{t-1}{2}\right)+1}{t^2}$ • $\frac{\frac{t^2+2t+1}{4}}{2\left(\frac{t-1}{2}\right)+1}$

Q10	Model Solution – 40 Marks	Marking Notes
(d) (iii)	**Method 1** $$\lim_{t\to\infty}\frac{t^2+2t+1}{4t^2}$$ $= \lim_{t\to\infty}\left(\frac{t^2}{4t^2}+\frac{2t}{4t^2}+\frac{1}{4t^2}\right)$ $= \lim_{t\to\infty}\left(\frac{1}{4}+\frac{1}{2t}+\frac{1}{4t^2}\right)$ $= \frac{1}{4}+0+0$ $= \frac{1}{4}$ **Method 2** $\lim_{t\to\infty}\frac{t^2+2t+1}{4t^2}$ $= \lim_{t\to\infty}\frac{\frac{t^2}{t^2}+\frac{2t}{t^2}+\frac{1}{t^2}}{\left(\frac{4t^2}{t^2}\right)}$ $= \lim_{t\to\infty}\frac{1+\frac{2}{t}+\frac{1}{t^2}}{4}$ $= \frac{1+0+0}{4}$ $= \frac{1}{4}$	**Scale 5C (0, 2, 3, 5)** Accept correct answer without work Consider solution as consisting of 3 steps: **Step 1.** Splits in **3** fractions / divides top and bottom by $\boldsymbol{t^2}$ **Step 2**. Simplifies (before taking limits) **Step 3**. Takes limits *Low Partial Credit:* • Work of merit, for example, substitutes 2 or more positive integers for t • Some correct differentiation *High Partial Credit:* • 2 steps correct • $\lim_{t\to\infty}\left(\frac{1+\frac{2}{t}+\frac{1}{t^2}}{4}\right)$ • $\lim_{t\to\infty}\frac{t^2}{4t^2}$

Q10	Model Solution – 40 Marks	Marking Notes
(e)	**(i)** $H(1) = 4$ **(ii)** $\boldsymbol{P(1)}$: $H(1) = (1+1)^2 = 4$ which is true $\boldsymbol{P(k)}$: Assume true for $n = k$, so $H(k) = (k+1)^2$ $\boldsymbol{P(k+1)}$: Prove for $n = k+1$ To prove: $H(k+1) = ((k+1)+1)^2$ $H(k+1) = H(k) + 2k + 3$ $= (k+1)^2 + 2k + 3$ [by $P(k)$] $= k^2 + 4k + 4$ $= (k+2)^2$ $= \left((k+1)+1\right)^2$. $\therefore P(k+1)$ is true [So true for $n = k+1$ if true for $n = k$.] Therefore, true for all $n \in \mathbb{N}$.	**Scale 15D (0, 4, 7, 10, 15)** Consider solution to **(i)** and **(ii)** combined as consisting of 4 steps: **Step 1.** $\boldsymbol{H(1)}$ identified in **(i) and** $\boldsymbol{P(1)}$ verified in **(ii)** **Step 2**. $\boldsymbol{P(k)}$ stated **Step 3**. $\boldsymbol{P(k+1)}$ stated **and** $\boldsymbol{H(k+1) = H(k) + 2k + 3}$ **Step 4**. $\boldsymbol{P(k+1)}$ proved Steps 1 and 2 can be in any order. *Low Partial Credit:* • Work of merit, for example, **(i)** correct, or $P(k+1)$ stated *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct *Full Credit –1:* • Omits part or all of conclusion but otherwise correct. Conclusion has three parts; these do not all have to come at the end of the proof: ○ $P(1)$ true ○ $P(k)$ true implies $P(k+1)$ true ○ $P(n)$ true for all $n \in \mathbb{N}$

Leaving Certificate 2025

Mathematics

Higher Level

Paper 2

Structure of the marking scheme – Paper 2

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled **A**, divide candidate responses into two categories (correct and incorrect), scales labelled **B** divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	**A**	**B**	**C**	**D**
Number of categories	2	3	4	5
5-mark scales		0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10-mark scales			0, 4, 6, 10	0, 3, 5, 7, 10
15-mark scale			0, 4, 8, 15	0, 4, 7, 10, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

General notes on the marking – Paper 2

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded. The only marks that may be awarded for a question are those on the scales above, or *Full Credit -1*.

Instructions regarding penalties for omitted or incorrect units are given in the scheme for each question to which they apply. A penalty for rounding is applied once per unit of marking (i.e., if parts (a) and (b) are marked as a single unit, a penalty for rounding is only applied once for (a) and (b) combined).

In general, an answer without sufficient supporting work is awarded the highest level of credit on the scale below *Full Credit* (typically *Partial Credit* or *High Partial Credit*, as appropriate).

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Steps
Where steps are listed in the Marking Notes, unless otherwise specified, it is to be taken that they can be independently correct / incorrect – that is, in a candidate's solution, step n can be considered correct even if previous step(s) have not been correctly presented, as long as the work done to arrive at step n from the previous step(s) has not been oversimplified. It is specifically noted where this does not hold. Note also that these steps may not need to be presented in the order specified in the Marking Notes.

Errors
Where a question is **not** marked using steps, if a candidate has a single error, they are generally awarded one level of credit below that which they would otherwise have been awarded. Similarly, where they have two errors, they are generally awarded two levels of credit below that which they would otherwise have been awarded. (If they present sufficient work for *Low Partial Credit*, they will be awarded this at a minimum, regardless of the number of errors.) For example, on a C-scale:

- *High Partial Credit*: One error, otherwise fully correct (or fully correct with a *)
- *Low Partial Credit*: Two errors, otherwise fully correct (or fully correct with a *)

Where a question **is** marked using steps, this does not apply. Instead, an error in a step means that the step has not been completed correctly; this does not affect the completion of other steps (unless it oversimplifies the work). So, if a candidate has multiple errors on a single step, they could still be awarded up to *High Partial Credit*, depending on the marking scheme.

The *
Where a candidate has a single * on their solution, this is ignored in the awarding of credit unless they would otherwise have *Full Credit*. Where a candidate has multiple *s, this is generally treated as an error.

Multiple answers
Where the solution requires substantial work, mark all separate attempts and award the marks for the best one, regardless of crossing out.

Where a solution requires selection from the question:
- If a candidate has crossed out answer(s), ignore the crossed-out answers
- If a candidate has multiple answers that are **not** crossed out, award the lowest mark associated with these answers (generally, this will be considered incorrect)

Square brackets
Where something is contained in square brackets in the model solution, it is **not** required for *Full Credit*.

Work of merit
Where the scheme indicates "work of merit", examples are given that exemplify the standard of work required to be considered work of merit in that particular question.

Palette of annotations available to examiners – Paper 2

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✓	Tick	Work of relevance	The work presented in the body of the script merits full credit
✗	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding / Unit / Arithmetic error / Misreading	
〰	Horizontal wavy	Error	
P	Partial Credit		The work presented in the body of the script merits *Partial Credit*
L	Low Partial Credit		The work presented in the body of the script merits *Low Partial Credit*
M	Mid Partial Credit		The work presented in the body of the script merits *Mid Partial Credit*
H	High Partial Credit		The work presented in the body of the script merits *High Partial Credit*
F*	F star		The work presented in the body of the script merits *Full Credit – 1*
[	Left Bracket		Another version of this solution is presented elsewhere, and it merits equal or higher credit
⌇	Vertical wavy	No work on this page / portion of this page	
O	Oversimplify	The candidate has oversimplified the work	
WOM	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and 〰 and 〰 appear in the body of the work, then L should be placed in the right margin.

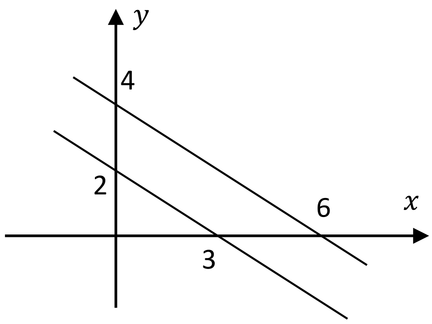
In the case of a **D scale** with the same annotations, M should be placed in the right margin.

Model Solutions & Marking Notes – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1** $3(p) - 2(5) + 28 = 0$ $3p + 18 = 0$ $3p = -18$ $p = -6$ **Method 2** $m = \frac{3}{2}$ and $(0, 14)$ is a point on the line $\frac{14 - 5}{0 - p} = \frac{3}{2}$ $-3p = 18$ $p = -6$	**Scale 5C (0, 2, 3, 5)** If equation is not solved algebraically, "supporting work" would be, for example, $(-6, 5)$ subbed into the equation and fully verified. Accept x used in place of p If 5 is substituted for x and p for y, award High Partial Credit at most. *Low Partial Credit:* • Work of merit, for example, 5 subbed in for y, or p subbed in for x • Relevant work to isolate p • Finds the slope of the line or another point on the line *High Partial Credit:* • One error, otherwise correct

<table>
<tr><th>Q1</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(b)</td><td>
Method 1
$m_l = -\frac{1}{3}$
$m_h = \frac{2}{5}$
$\tan\theta = \pm\frac{-\frac{1}{3}-\frac{2}{5}}{1+\left(-\frac{1}{3}\right)\left(\frac{2}{5}\right)}$
$\tan\theta = \pm\frac{-11}{13}$
$\theta = 40{\cdot}2\ldots^\circ = 40[^\circ]\ \ [\in \mathbb{N}]$
Method 2
$m_l = -\frac{1}{3}$
$m_h = \frac{2}{5}$
$\tan^{-1}\left(-\frac{1}{3}\right) = 161\cdot 56\ldots^0$
$\tan^{-1}\left(\frac{2}{5}\right) = 21\cdot 80\ldots^0$
$\alpha = 161\cdot 56\ldots^0 - 21\cdot 80\ldots^0$
$= 139\cdot 76\ldots^\circ$
$\theta = 180^0 - 139\cdot 76\ldots^0$
$= 40{\cdot}24\ldots^\circ$
$= 40[^\circ]\ \ [\in \mathbb{N}]$
Method 3
$\theta = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{5}$
$= 40\cdot 2\ldots$
$= 40[^\circ]\ \ [\in \mathbb{N}]$
</td><td>
Scale 10D (0, 3, 5, 7, 10)
Accept correct answer without unit.
Consider solution as consisting of 4 steps:
Method 1
Note: For Step 3, accept substitution without $\pm$
Step 1. Finds m_l
Step 2. Finds m_h
Step 3. Subs in formula
Step 4. Finds θ
Method 2
Step 1. Finds m_l
Step 2. Finds m_h
Step 3. Finds $161\cdot 56\ldots^0$ and $21\cdot 8\ldots^0$
Step 4. Finds θ
Method 3
Step 1. Finds m_l
Step 2. Finds m_h
Step 3. $\theta = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{5}$
Step 4. Finds θ
Low Partial Credit:
• 1 step correct
• Some correct work towards rearranging h
• Diagram showing a graph of the two lines
Mid Partial Credit:
• 2 steps correct
High Partial Credit:
• 3 steps correct
Full Credit –1:
• Finds both acute and obtuse angles, doesn’t specify which is the answer
• Apply a * for incorrect rounding.
</td></tr>
</table>

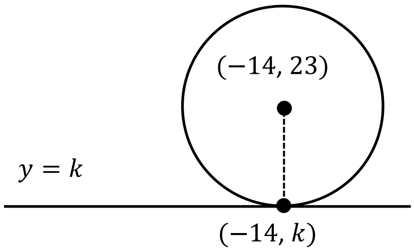
Q1	Model Solution – 30 Marks	Marking Notes
(c)	**Method 1:** [First 2 possible lines AB in 1st quadrant:]  $a = 3, b = 2$: area $= \frac{1}{2}(2)(3) = 3$ $a = 6, b = 4$: area $= \frac{1}{2}(4)(6) = 12.$ So $a = 6, b = 4$, **or** $a = -6, b = -4$. *Equations:* $y = -\frac{2}{3}x + 4$ **and** $y = -\frac{2}{3}x - 4$ $[2x + 3y = 12$ **and** $2x + 3y = -12]$ **Method 2:** Area: $\frac{1}{2}ab = 12$ $ab = 24$ $a = \frac{24}{b}$ *Slope:* $-\frac{b}{a} = -\frac{2}{3}$ $2a = 3b$ $2\left(\frac{24}{b}\right) = 3b$ $48 = 3b^2$ $b^2 = 16$ $b = \pm 4$ *Equations:* $y = -\frac{2}{3}x + 4$ and $y = -\frac{2}{3}x - 4$	**Scale 15D (0, 4, 7, 10, 15)** Note: If **both** the slope and the area are not used to find a and b, then no credit can be awarded for finding the equations of the lines. Consider the solution as consisting of 4 steps: **Method 1** If Step 2 is done, consider Step 1 to be done as well. **Step 1**. Finds area for one set of a and b (not a solution) **Step 2**. Finds area for a correct set of values of a and b **Step 3**. Finds one equation **Step 4**. Finds second equation **Method 2** **Step 1**. 1 equation in a and b **Step 2**. Second equation in a and b **Step 3**. Finds a **or** b **Step 4**. Finds equations of 2 lines *Low Partial Credit:* • Work of merit, for example, relevant diagram drawn (line with negative slope), or some correct substitution in area formula or equation of a line formula • Plots $(0, b)$, where $b \in 2k, k \in \mathbb{Z}\backslash\{0\}$ • Plots $(a, 0)$ where $a \in 3k, k \in \mathbb{Z}\backslash\{0\}$ *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct • The equation of one line found

Q2	Model Solution – 30 Marks	Marking Notes
(a) (i)	Centre $= (4, -2)$ Radius $= \sqrt{45}$ **or** $3\sqrt{5}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, identifies h, k, or r^2 • $(-4, 2)$ and no further work *High Partial Credit* • Centre or radius correct *Full Credit -1:* • Radius given as a decimal, otherwise correct

Q2	Model Solution – 30 Marks	Marking Notes
(ii)	**Method 1** Slope from $(4,-2)$ to $(-2,-5)$: $m = \frac{-5+2}{-2-4} = \frac{-3}{-6} = \frac{1}{2}$ *Slope of tangent line* $= -2$ *Equation:* $y-(-5) = -2(x-(-2))$ $y+5 = -2x-4$ $y = -2x-9$ **Method 2*** *Equation of circle:* $x^2+y^2-8x+4y-25=0$ *Equation of tangent:* $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$ $-2x-5y-4(x-2)+2(y-5)-25=0$ $y=-2x-9$ **Method 3*** *Equation of tangent:* $(x-h)(x_1-h)+(y-k)(y_1-k) = r^2$ $(x-4)(-2-4)+(y+2)(-5+2)=45$ $y=-2x-9$ **Method 4** $x^2-8x+16+y^2+4y+4=45$ $2x-8+2y\frac{dy}{dx}+4\frac{dy}{dx}=0$ $(2y+4)\left(\frac{dy}{dx}\right)=-2x+8$ $\frac{dy}{dx}=\frac{-2x+8}{2y+4}$ *Slope of tangent line:* $\frac{dy}{dx}(-2,-5)=\frac{-2(-2)+8}{2(-5)+4}$ $=-2$ $y+5=-2x-4$ $y=-2x-9$ *Candidates are not expected to be familiar with methods 2 or 3, but these methods will nonetheless be accepted.	**Scale 10D (0, 3, 5, 7, 10)** Consider solution as consisting of 4 steps: **Method 1** Step 3 is not correct if Step 2 is not completed (with or without errors) **Step 1**. Finds slope of the normal **Step 2**. Finds slope of the tangent **Step 3**. Substitutes values into formula for equation of a line **Step 4**. Equation in required form **Method 2** **Step 1**. Finds g and f **Step 2**. Finds c **Step 3**. Substitutes values into formula **Step 4**. Equation in required form **Method 3** **Step 1**. Identifies r^2 **Step 2**. Finds h and k **Step 3**. Substitutes values into formula **Step 4**. Equation in required form **Method 4** **Step 1**. Finds $\frac{dy}{dx}$ in terms of x and y **Step 2**. Finds slope of the tangent **Step 3**. Substitutes values into formula **Step 4**. Equation in required form *Low Partial Credit:* • Work of merit, for example, diagram drawn with tangent line shown, or indicates perpendicular • Some correct substitution into formula for the equation of a line or the formula for the equation of a tangent • Some correct differentiation *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct *Full Credit -1:* • y isolated and correct but not in correct form

<table>
<tr><th>Q2</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(b)</td><td>
Method 1:

Centre: $(-14, 23)$



Point of tangency on circle: $(-14, k)$

$(-14)^2 + k^2 + 28(-14) - 46k + k = 0$ ...①

$k^2 - 45k - 196 = 0$

$(k - 49)(k + 4) = 0$
$k = 49$ or $k = -4$

Method 2:

Centre $= (-14, 23)$ Radius $= \sqrt{725 - k}$

Tangent: $0 + y - k = 0$

Perp distance tangent to centre = radius:
$\frac{|0+1(23)-k|}{\sqrt{0^2+1^2}} = \sqrt{725 - k}$ ... ①

$(23 - k)^2 = 725 - k$

$529 - 46k + k^2 - 725 + k = 0$

$k^2 - 45k - 196 = 0$

$(k - 49)(k + 4) = 0$

$k = 49$ or $k = -4$

Method 3

Radius $= \sqrt{725 - k}$ Centre $= (-14, 23)$

$23 + \sqrt{725 - k} = k$
$725 - k = (k - 23)^2$
$k^2 - 45k - 196 = 0$

$(k - 49)(k + 4) = 0$

$k = 49$ or $k = -4$
</td><td>
Scale 15D (0, 4, 7, 10, 15)

Consider solution as consisting of 4 steps:

Method 1
Step 1. Finds in terms of k, point of tangency
Step 2. Equation in k (① in solution)
Step 3. Quadratic equation in the form $k^2 - 45k - 196 = 0$ or equivalent.
Step 4. Finds k

Method 2
Step 1. Finds centre and radius
Step 2. Equation in k (① in solution)
Step 3. Quadratic equation in form $k^2 - 45k - 196 = 0$ or equivalent.
Step 4. Finds k

Method 3
Step 1. Finds centre and radius
Step 2. $23 + \sqrt{725 - k}$ or $23 - \sqrt{725 - k}$
Step 3. Quadratic equation in form $k^2 - 45k - 196 = 0$ or equivalent.
Step 4. Finds k

Low Partial Credit:
• Work of merit, for example, finds centre or radius, or draws diagram with horizontal tangent

Mid Partial Credit:
• 2 steps correct

High Partial Credit:
• 3 steps correct
</td></tr>
</table>

<table>
<tr><th>Q3</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)(i)</td><td>$P(A) = \dfrac{23+18+6+13}{240}$
$= \dfrac{60}{240}$
$= \dfrac{1}{4}$</td><td>Scale 5B (0, 2, 5)

Accept $\frac{60}{240} = \frac{1}{4}$ for full credit

Partial Credit:
• Work of merit, for example, finds one relevant probability, such as, $\frac{23}{240}$; or adds relevant numbers, for example, $23 + 18$ (but with no irrelevant numbers added)

Full Credit -1:
• $\frac{60}{240}$ and stops</td></tr>
<tr><td>(a)(ii)</td><td>$P(A \cup C) = \dfrac{60+16+41}{240}$
$= \dfrac{117}{240}$

$P(A) = \frac{60}{240}$

$P(C) = \frac{76}{240}$

$P(A \cap C) = \frac{19}{240}$

$P(A) + P(C) - P(A \cap C) = \dfrac{60+76-19}{240}$
$= \dfrac{117}{240}$
$= [P(A \cup C)]$</td><td>Scale 5C (0, 2, 3, 5)

Consider solution as requiring 4 steps:
Step 1. Find $P(A \cup C)$
Step 2. Find $P(C)$
Step 3. Find $P(A \cap C)$
Step 4. Substitute in $P(A) + P(C) - P(A \cap C)$

Low Partial Credit:
• Work of merit, for example, one of $\#A, \#C, \#(A \cup C)$
• 1 step correct

High Partial Credit:
• 3 steps correct</td></tr>
</table>

Q3	Model Solution – 30 Marks	Marking Notes
(a)(iii)	**Answer**: YES **Justification**: **Method 1** [Does $P(A \cap B) = P(A) \times P(B)$?] $P(A \cap B) = \frac{18+6}{240}$ $= \frac{24}{240}$ $= \frac{1}{10}$ $P(A) = \frac{1}{4}$ and $P(B) = \frac{96}{240} = \frac{2}{5}$ $P(A) \times P(B) = \frac{1}{4} \times \frac{2}{5}$ $= \frac{1}{10}$ $= [P(A \cap B)]$ **Method 2** [Is $P(A\|B) = P(A)$?] $P(A\|B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{24}{96}$ $= \frac{1}{4}$ $= [P(A)]$ **Method 3** [Is $P(B\|A) = P(B)$?] $P(B) = \frac{18+6+16+56}{240}$ $= \frac{2}{5}$ $P(B\|A) = \frac{P(B \cap A)}{P(A)}$ $= \frac{18+6}{60}$ $= \frac{2}{5}$ $= [P(B)]$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • Answer correct • Work of merit in justification, for example, states a relevant condition, or finds a relevant probability *Mid Partial Credit:* • Answer correct **and** work of merit in justification • Incorrect or no answer, but substantial work of merit in justification, for example, states a relevant condition and finds 2 relevant probabilities *High Partial Credit:* • Answer correct **and** substantial work of merit in justification, for example, states a relevant condition and finds 2 relevant probabilities • Justification sufficient to support the correct answer, but no or incorrect answer

Q3	Model Solution – 30 Marks	Marking Notes
(b)	**Method 1** $P(A \cap B \cap C) = \frac{6}{240}$ $P(\text{none}) = \frac{67}{240}$ $P(\text{both}) = 2\left[\left(\frac{6}{240}\right) \times \left(\frac{67}{239}\right)\right]$ $= \frac{402}{28680}$ $= \left[\frac{67}{4780}\right]$ **Method 2** $\frac{\binom{6}{1}\binom{67}{1}}{\binom{240}{2}} = \frac{67}{4780}$	**Scale 15D (0, 4, 7, 10, 15)** *Low Partial Credit:* • Finds one relevant probability, for example, $\frac{6}{240}$ • Writes one of $\binom{6}{1}, \binom{67}{1}, \binom{240}{2}$ • $\frac{67}{240} \times \frac{6}{240}$ not evaluated *Mid Partial Credit:* • $\frac{6}{240} \times \frac{67}{239}$ or similar not evaluated • $2\left(\frac{67}{240} \times \frac{6}{240}\right)$ not evaluated • $\frac{\binom{6}{1}}{\binom{240}{2}}$ or $\frac{\binom{67}{1}}{\binom{240}{2}}$ *High Partial Credit:* • $2\left(\left(\frac{6}{240}\right) \times \left(\frac{67}{239}\right)\right)$ • $\frac{\binom{6}{1}\binom{67}{1}}{\binom{240}{2}}$

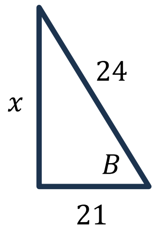
Q4	Model Solution – 30 Marks	Marking Notes
(a)	**(i)** Median$=\frac{x+18}{2}$ $\frac{x+18}{2}=17\cdot5$ $x+18=35$ $x=17$ **(ii)** $Q_3=\frac{19+22}{2}$ $=20\cdot5$ $IQR=Q_3-Q_1$ $=20\cdot5-13$ $=7\cdot5$	**Scale 15D (0, 4, 7, 10, 15)** Accept correct answer without work in **(i)** *Low Partial Credit:* • Work of merit, for example, for **(i)**, shows median as middle number in the list, or, for **(ii)**, states $IQR=Q_3-Q_1$ or work towards finding Q_3 • Relevant work on the diagram *Mid Partial Credit:* • 1 part correct • Work of merit in both parts *High Partial Credit:* • 1 part correct **and** work of merit in the other part
(b)	Answer: Mean only Justification: *Two parts are required: why the mean changes and why the median does not.* The total will increase, so the mean will increase as well *(needs to refer to the sum / total, or similar, increasing to be fully correct)* The median only depends on 5th and 6th biggest numbers, these won't change if biggest number increases *(needs to refer to middle, or similar, to be fully correct)*	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Answer correct • Justification shows understanding of effect on median **or** mean *High Partial Credit:* • Answer correct **and** justification correct regarding one of the statistics (median **or** mean) • Justification sufficient to support the correct answer, but no or incorrect answer *Full Credit –1:* • Apply a * if the justification is a particular example rather than a more general justification.

Q4	Model Solution – 30 Marks	Marking Notes
(c)	$\frac{27(4)+33(5)+39(9)+45(k)+51(4)+57(2)}{4+5+9+k+4+2} = 40{\cdot}4$ $\frac{942+45k}{k+24} = 40{\cdot}4$ $942 + 45k = 40.4k + 969.6$ $4.6k = 27.6$ $k = 6$	**Scale 10D (0, 3, 5, 7, 10)** Note: If k does not appear in both the numerator and denominator, then *Mid Partial Credit* at most No penalty if data is assumed to be integer-valued, i.e., intervals treated as $24-29, 30-35$, and so on Consider solution as consisting of 4 steps: **Step 1**. Finds mid-interval values **Step 2**. Finds a correct expression in k for either the numerator or denominator **Step 3**. Writes an equation in k **Step 4**. Solves for k *Low Partial Credit:* • Work of merit, for example, 1 correct mid-interval value identified, or multiplies a "number of people" by a relevant age (for example, min or max in the class) • Correct answer with no supporting work *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct *Full Credit -1:* • Apply a * for one incorrect MIV, otherwise correct

Q5	Model Solution – 30 Marks	Marking Notes
(a)	**S1.** $\|AC\| = \|BC\|$ ... C is midpoint of $[AB]$ **S2.** $\|\angle DCA\| = \|\angle BCE\|$ ... vertically opposite **S3.** $\|\angle DAC\| = \|\angle CBE\|$ ... alternate angles **S4.** So $ACD \equiv BCE$ by ASA. **OR** **S1.** $\|AC\| = \|BC\|$ ... C is midpoint of $[AB]$ **S2.** $\|\angle DCA\| = \|\angle BCE\|$ ... vertically opposite **S3.** $\|\angle ADC\| = \|\angle CEB\|$ ... alternate angles **S4.** So $ACD \equiv BCE$ by AAS. **OR** **S1.** $\|AC\| = \|BC\|$ ... C is midpoint of $[AB]$ **S2.** $\|\angle DAC\| = \|\angle CBE\|$ ... alternate angles **S3.** $\|\angle ADC\| = \|\angle CEB\|$ ... alternate angles **S4.** So $ACD \equiv BCE$ by AAS.	**Scale 15D (0, 4, 7, 10, 15)** Note: For S1 – S3, accept 2 correct reasons. Work indicated on the diagram is generally taken as a statement without a reason *Low Partial Credit:* • Work of merit, for example, a relevant fact indicated (text or on diagram) *Mid Partial Credit:* • 2 statements from S1, S2, S3 (reasons not required) *High Partial Credit:* • 3 statements with reasons *Full Credit:* • 4 statements with reasons. *Full Credit -1:* • If AAS is established, accept S4 if ASA or AAS given as reason
(b)(i)	$k = \frac{\|XQ'\|}{\|XQ\|}$ $= \frac{12}{8}$ $= 1 \cdot 5$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, states $k = \frac{\|XQ'\|}{\|XQ\|}$ or finds $\|XQ'\|$

Q5	Model Solution – 30 Marks	Marking Notes
(b)(ii)	**Method 1** $\lvert XP'\rvert = 1.5 \times 3$ $= 4\cdot 5$ So, $\lvert P'Q\rvert = 3{\cdot}5$ Area $= \dfrac{\lvert P'Q\rvert}{\lvert PQ\rvert} \times 20$ $= \dfrac{3\cdot 5}{5} \times 20$ $= 14[\text{cm}^2]$ **Method 2** $\lvert PQ\rvert = 8 - 3 = 5$ $\lvert P'Q'\rvert = 5 \times 1{\cdot}5$ $= 7\cdot 5$ $\lvert P'Q\rvert = 7{\cdot}5 - 4$ $= 3\cdot 5$ Area $= \dfrac{\lvert P'Q\rvert}{\lvert PQ\rvert} \times 20$ $= \dfrac{3\cdot 5}{5} \times 20$ $= 14[\text{cm}^2]$ **Method 3** $\lvert PQ\rvert = 8 - 3 = 5$ $\lvert PP'\rvert = \lvert XP'\rvert - \lvert XP\rvert$ $= 3(1{\cdot}5) - 3$ $= 1{\cdot}5$ Area $PQRS$= $\lvert PQ\rvert\lvert PS\rvert \sin\angle SPQ$ $= 5\lvert PS\rvert \sin\angle SPQ = 20$ $\therefore\ \lvert PS\rvert \sin\angle SPQ = 4$ Area $P'QRY$ = Area $PQRS$ − Area $PP'YS$ $= 20 - 1{\cdot}5\lvert PS\rvert \sin\angle SPQ$ $= 20 - 1{\cdot}5(4)$ $= 14[\text{cm}^2]$	**Scale 10D (0, 3, 5, 7, 10)** Accept correct answer without unit. Consider solution as consisting of 4 steps: **Methods 1&2:** **Step 1**. Finds one relevant length **Step 2**. Finds $\lvert P'Q\rvert$ **Step 3**. Indicates $\frac{\lvert P'Q\rvert}{\lvert PQ\rvert}$ **Step 4**. Finds area **Method 3:** **Step 1**. Finds one relevant length **Step 2**. Finds $\lvert PP'\rvert$ **Step 3**. Shows $\lvert PS\rvert\, sin(\angle\, SPQ) = 4$ or equivalent **Step 4**. Finds area *Low Partial Credit:* • 1 step correct *Mid Partial Credit:* • 2 steps correct • Finds $h = 4$ *High Partial Credit:* • 3 steps correct

Q6	Model Solution – 30 Marks	Marking Notes
(a)	$\sin^{-1}\left(\frac{1}{2}\right) = 30°$ 1st and 2nd quadrants: $A = 30°$ or $A = 180 - 30°$ $\quad = 150^0$ $-360°: A = -330°$ or $-210°$ $+360°: A = 390°$ or $510°$ $\{-330^0, -210^0, 30^0, 150^0, 390^0, 510^0\}$	**Scale 5D (0, 2, 3, 4, 5)** Accept correct answers without work Consider solution as consisting of 4 steps: **Step 1**. Finds reference angle **Step 2**. Finds 2nd angle in first revolution **Step 3**. Finds values $< 0°$ **Step 4**. Finds values $> 360°$ *Low Partial Credit:* • Work of merit, for example, indicates the quadrant in which A lies *Mid Partial Credit:* • 2 steps correct • Any three correct angles found *High Partial Credit:* • 3 steps correct • 4 angles correct including 30^0 and 150^0 *Full Credit –1:* • Apply a * once for excess values • Apply a * once for one or more values given in radians
(b)	**Period** $= \frac{1}{2}(2\pi) = \pi$ [radians] **Range**: $\lvert\sin x\rvert$ has a range of $[0, 1]$ $\lvert 4\sin x\rvert$ has a range of $[0, 4]$ $\lvert 4\sin x\rvert - 1$ has a range of $[-1, 3]$	**Scale 15D (0, 4, 7, 10, 15)** Accept correct answers without work. Accept correct period without unit. *Low Partial Credit:* • Work of merit, for example, for period, mentions 2π, or, for range, indicates correct y-value on the graph (-1 or 3) *Mid Partial Credit:* • Period or range correct • Work of merit for both *High Partial Credit:* • Period or range correct **and** work of merit for the other. *Full Credit -1:* • Period given in degrees

Q6	Model Solution – 30 Marks	Marking Notes
(c)	$(2)^2 = (3)^2 + (4)^2 - 2(3)(4)\cos B$ $\cos B = \dfrac{25-4}{24}$ $= \dfrac{21}{24}$ $24^2 = x^2 + 21^2$ $x^2 = 135$ $x = \sqrt{135}$ $= 3\sqrt{15}$  $\tan \angle CBA = \dfrac{3\sqrt{15}}{21}$ $= \dfrac{\sqrt{15}}{7}$	**Scale 10D (0, 3, 5, 7, 10)** Consider the solution as consisting of 3 steps: **Step 1**. Fills in cosine rule **Step 2**. Finds $cos\ \angle CBA$ **Step 3**. Finds $tan\ \angle CBA$ The theorem of Pythagoras must be used in Step 3, for Step 3 to be considered correct *Low Partial Credit:* • Work of merit, for example, some correct substitution into the cosine rule *Mid Partial Credit:* • 1 step correct *High Partial Credit:* • 2 steps correct

Q7	Model Solution – 50 Marks	Marking Notes
(a)	**(i)** $r = 1$ [cm]; $d = 10$ [cm]; $h = 3$ [cm] **(ii)** $r = \frac{1}{14} \times 90 = 6{\cdot}42 \ldots = 6{\cdot}4$ [m] [1 D.P.] $d = \frac{10}{14} \times 90 = 64{\cdot}28 \ldots = 64{\cdot}3$ [m] [1 D.P.] $h = \frac{3}{14} \times 90 = 19{\cdot}28 \ldots = 19{\cdot}3$ [m] [1 D.P.]	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit:* • Work of merit, for example, in **(i)**, 1 value correct, or in **(ii)**, finds 14 *Mid Partial Credit:* • 1 part correct (**(i)** or **(ii)**) *High Partial Credit:* • 1 part correct **and** work of merit in the other part *Full Credit –1:* • Apply a * once for early rounding in **(ii)** (uses 6·4 to find the other two lengths)
(b)	$\frac{2}{3}\pi x^3 + \pi x^2(7x) + \frac{1}{3}\pi x^2(4x) = 6738$ …① $9\pi x^3 = 6738$ $x^3 = \dfrac{6738}{9\pi}$ $x = \sqrt[3]{\dfrac{6738}{9\pi}}$ $= 6 \cdot 19 \ldots$ $\text{Total length} = 12(6 \cdot 19 \ldots)$ $= 74 \cdot 39 \ldots$ $= 74 \cdot 4 \text{ [m][1 D.P.]}$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • x correctly filled into one relevant formula (including volume of a sphere) • Equation in x *High Partial Credit:* • Isolates x^3 *Full Credit -1:* • x found correctly and no further work

Q7	Model Solution – 50 Marks	Marking Notes
(c)(i) **(ii)**	**(i)** $(x-20)^2+y^2=178$ **(ii)** $(7-20)^2+(3)^2=178$ $169+9=178\ldots\text{True}$	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit:* • Work of merit, for example, in **(i)**, identifies centre or radius, or, in **(ii)**, some correct substitution into the equation of the circle *Mid Partial Credit:* • 1 part correct (**(i)** or **(ii)**) • Work of merit in both parts *High Partial Credit:* • 1 part correct **and** work of merit in the other part • Circle k is used instead of s, otherwise correct
(c)(iii)	**Method 1** $\frac{1}{2}(6)(20-7)=39$ [km²] **Method 2** $(20,0)\rightarrow(0,0)$ $(7,3)\rightarrow(-13,3)$ $(7,-3)\rightarrow(-13,-3)$ $\frac{1}{2}\lvert(-13)(-3)-(-13)(3)\rvert=39$ [km²]	**Scale 5C (0, 2, 3, 5)** Accept correct answer without unit *Low Partial Credit:* • Work of merit, for example, some correct substitution into the formula for the area of a triangle, or indicates a relevant translation *High Partial Credit:* • Fully substituted formula

Q7	Model Solution – 50 Marks	Marking Notes
(c)(iv)	**Method 1** $\lvert\angle CBD\rvert = 2\tan^{-1}\left(\frac{3}{13}\right)$ $= 25 \cdot 9 \ldots$ $= 26\ [°][\in \mathbb{N}]$ **Method 2** $\frac{1}{2}(\sqrt{178})(\sqrt{178})\sin\angle CBD = 39$ $\sin\angle CBD = \frac{78}{178}$ $\lvert\angle CBD\rvert = \sin^{-1}\left(\frac{78}{178}\right)$ $\lvert\angle CBD\rvert = 25{\cdot}9 \ldots$ $= 26[°][\in \mathbb{N}]$ **Method 3** $6^2 = 178 + 178 - 2(178)\cos\angle CBD$ $\cos\angle CBD = \frac{320}{356}$ $\lvert\angle CBD\rvert = 25 \cdot 9 \ldots$ $= 26[°][\in \mathbb{N}]$	**Scale 5C (0, 2, 3, 5)** Accept correct answer without unit Note: Lengths indicated on the diagram are not awarded credit in this part. (They may be awarded credit in earlier parts.) Note: If an incorrect answer in (c)(iii) produces an invalid triangle, then correct work in this part will lead to a contradiction ($sin\,\angle CBD > 1$). This can be awarded *High Partial Credit* at most. *Low Partial Credit:* • Work of merit, for example, identifies opposite or adjacent • Some correct substitution in the formula, $A = \frac{1}{2}ab\sin C$ • Some correct substitution in the cosine rule formula *High Partial Credit:* • Finds $\lvert\angle CBA\rvert$ (half of required angle) • $2\tan^{-1}\left(\frac{3}{13}\right)$ • $\sin B = \frac{78}{178}$ • $\cos\angle CBD = \frac{320}{356}$ *Full Credit –1:* • Apply a * for calculator in an incorrect mode • Apply a * for incorrect rounding

Q7	Model Solution – 50 Marks	Marking Notes
(c)(v)	**Method 1** $$\text{Area sector } CBD = \frac{26}{360}\pi\left(\sqrt{178}\right)^2$$ $$= 40 \cdot 3869 \ldots$$ $$\text{Sector} - \Delta CBD = 40{\cdot}3869 \ldots - 39$$ $$= 1{\cdot}3869 \ldots$$ $$\text{Sector} - \Delta ADC = 23{\cdot}4837 - 21$$ $$= 2{\cdot}4837$$ $$\text{Total shaded area} = 1{\cdot}3869 \ldots + 2{\cdot}4837 \ldots$$ $$= 3 \cdot 87[\text{km}^2][2\text{ D.P.}]$$ **Method 2** $$\text{Area sector } CBD = \frac{26}{360}\pi\left(\sqrt{178}\right)^2$$ $$= 40 \cdot 3869 \ldots$$ Total shaded area: $$40{\cdot}3869 \ldots + 23{\cdot}4837 - 39 - 21$$ $= 3{\cdot}87$ [km^2] [2 D.P.] **Method 3** $$\lvert\angle CBD\rvert = \frac{13\pi}{90}$$ $$\text{Area of sector } CBD = \frac{1}{2}\left(\sqrt{178}\right)^2\frac{13\pi}{90}$$ $$= 40{\cdot}3869 \ldots$$ Total shaded area: $$40{\cdot}3869 \ldots + 23{\cdot}4837 - 39 - 21$$ $= 3{\cdot}87$ [km^2] [2 D.P.]	**Scale 10D (0, 3, 5, 7, 10)** Consider the solution as requiring 4 steps: **Step 1**. Fills in formula for sector CBD **Step 2**. Finds area of sector CBD **Step 3**. Finds one area (sector – triangle) **or** sums areas of sectors **or** sums areas of triangles **Step 4**. Finds required area *Low Partial Credit:* • Work of merit, for example, some correct substitution into formula for area of a circle or sector or finds the area of (sector-triangle ADC). *Mid Partial Credit:* • 2 steps correct *High Partial Credit* • 3 steps correct

Q8	Model Solution – 50 Marks	Marking Notes
(a)(i)	$\lvert OB\rvert^2 + \lvert OB\rvert^2 = 6^2$ $2\lvert OB\rvert^2 = 36$ $\lvert OB\rvert^2 = 18$ $\lvert OB\rvert = 3\sqrt{2}$ [m] **OR** $\lvert OB\rvert^2 = 3^2 + 3^2$ $= 18$ $\lvert OB\rvert = 3\sqrt{2}$ [m] $(3\sqrt{2})^2 + \lvert OP\rvert^2 = 11^2$ $\lvert OP\rvert^2 = 103$ $\lvert OP\rvert = \sqrt{103}$ [m]	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit:* • Work of merit in one part, for example, correct substitution into Pythagoras' Theorem or finds $\lvert OB\rvert$ without using Pythagoras *Mid Partial Credit:* • 1 part correct ($\lvert OB\rvert$ or $\lvert OP\rvert$) • Work of merit in both parts *High Partial Credit:* • 1 part correct **and** work of merit in the other part *Full Credit –1:* • Apply a * for incorrect or no unit in $\lvert OP\rvert$
(a)(ii)	**Method 1** Area 1 face $= \left(\frac{1}{2}\right)(6)(11)(\sin 74{\cdot}2)$ $= 31 \cdot 75 \ldots$ Area 4 faces $= 4 \times 31 \cdot 75 \ldots$ $= 127[\text{m}^2][\in \mathbb{N}]$ **Method 2** Area 1 face $= \left(\frac{1}{2}\right)(11)(11)(\sin 31{\cdot}6)$ $= 31 \cdot 71 \ldots$ Area 4 faces $= 4 \times 31 \cdot 71 \ldots$ $= 127[\text{m}^2][\in \mathbb{N}]$ **Method 3** $\tan 74.2^0 = \frac{h}{3}$ $h = 3\tan 74 \cdot 2^0$ Area 1 face $= \left(\frac{1}{2}\right)(6)(3\tan 74 \cdot 2^0)$ $= 31 \cdot 80 \ldots$ Area 4 faces $= 4 \times 31 \cdot 80 \ldots$ $= 127[\text{m}^2][\in \mathbb{N}]$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, diagram drawn with at least 2 correct values filled in *High Partial Credit:* • Finds area of 1 face • One error in finding area of 1 face, but finishes correctly • $4 \times \left(\frac{1}{2}\right)(11)(11)(\sin 31{\cdot}6)$ or equivalent

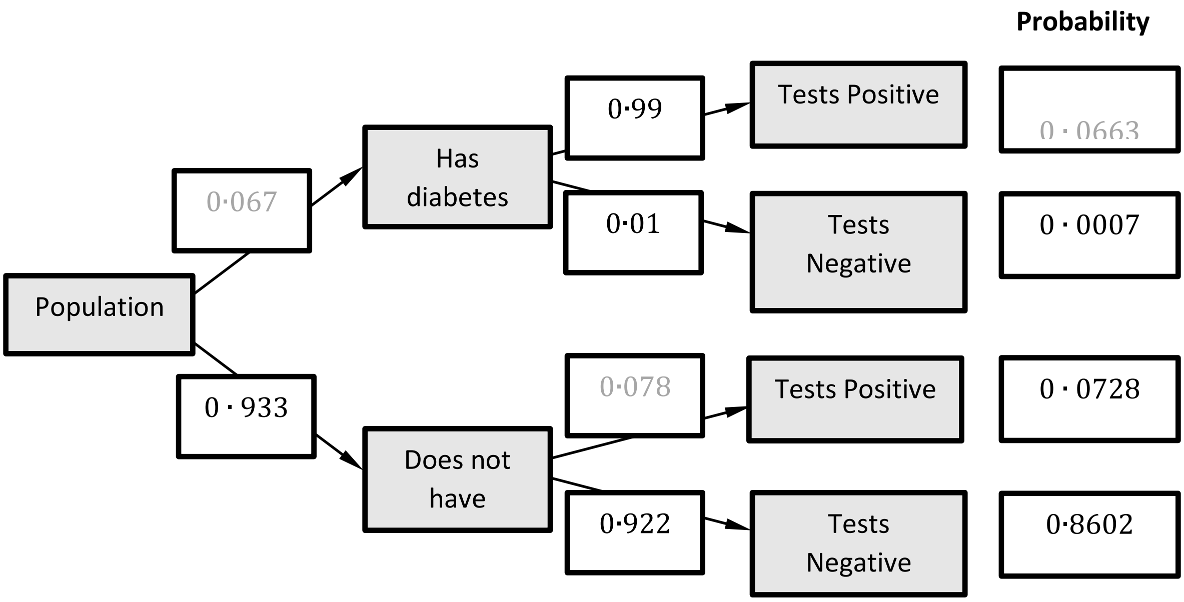
Q8	Model Solution – 50 Marks	Marking Notes
(a)(iii)	*Any valid net – note that two faces should not coincide when the solid is made.*	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, a sketch drawn of one triangle in the correct position *High Partial Credit:* • 1 triangle constructed correctly (construction lines visible) *Full Credit -1:* • Construction lines missing, otherwise correct
(b)	$\tan A = \dfrac{17{\cdot}5}{15}$ $A = \tan^{-1}\left(\dfrac{17{\cdot}5}{15}\right)$ $= 49 \cdot 39 \ldots$ % Error $= \frac{52-49{\cdot}39\ldots}{49{\cdot}4} \times 100$ $= \frac{2{\cdot}60\ldots}{49{\cdot}4} \times 100$ $= 5{\cdot}26 \ldots$ $= 5{\cdot}3$ [%] [1 DP]	**Scale 10C (0, 4, 6, 10)** Note: The % error may be calculated correctly based on an incorrect A *Low Partial Credit:* • Work of merit, for example, opposite and adjacent identified on the diagram • Either A or % Error calculated correctly *High Partial Credit:* • A correct and work of merit in calculating % Error • % Error correct and work of merit in calculating A • Answer not given as a percentage *Full Credit-1:* • Calculator in an incorrect mode, otherwise correct • Incorrect or no rounding

Q8	Model Solution – 50 Marks	Marking Notes
(c)(i)	*Angle opposite* x: $180 - 35 = 145°$ *Angle opposite* 10: $35 - 22 = 13°$ *Sine rule:* $\dfrac{x}{\sin 145} = \dfrac{10}{\sin 13}$ $x = \dfrac{10 \sin 145^0}{\sin 13^0}$ $= 25.5[\text{m}][1\text{ D.P.}]$	**Scale 10D (0, 3, 5, 7, 10)** Consider the solution as consisting of 4 steps: **Step 1**. Finds angle opposite 10 m **Step 2**. Finds angle opposite x **Step 3**. Sine Rule fully substituted **Step 4**. Shows $x = 25 \cdot 5$ Note: If substitutions in Step 3 oversimplify the work, then Step 4 is not considered correct Note: If $x \neq 25.5$ in Step 4, then a statement stating that $x \neq 25.5$ is required *Low Partial Credit:* • Work of merit, for example, some correct substitution in the Sine rule *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct *Full Credit -1:* • Calculator in an incorrect mode

Q8	Model Solution – 50 Marks	Marking Notes
(c)(ii)	**Method 1** Let $y = h - 1 \cdot 25$ $\sin 22^0 = \dfrac{y}{25 \cdot 5}$ $y = 25 \cdot 5 \times \sin 22^0$ $= 9 \cdot 55 \ldots$ $h = 9 \cdot 55 \ldots + 1 \cdot 25$ $= 10 \cdot 80 \ldots$ $= 10 \cdot 8\ [\mathrm{m}][1\ \mathrm{D.P.}]$ **Method 2** Let p be the length of the side opposite 22^0 $\dfrac{10}{\sin 13^0} = \dfrac{p}{\sin 22^0}$ $p = 16.652 \ldots$ Let $y = h - 1 \cdot 25$ $\sin 35^0 = \dfrac{y}{16.652 \ldots}$ $y = 9 \cdot 551 \ldots$ $h = 9 \cdot 551 \ldots + 1 \cdot 25$ $= 10 \cdot 80 \ldots$ $= 10 \cdot 8\ [\mathrm{m}][1\ \mathrm{D.P.}]$	**Scale 10D (0, 3, 5, 7, 10)** Accept correct answer without unit. Consider solution as consisting of 3 steps: **Step 1**. Sets up equation in y $[h - 1{\cdot}25]$ **Step 2**. Finds y **Step 3**. Finds h Step 3 is not considered correct, if no work is presented on Step 2. *Low Partial Credit:* • Work of merit, for example, extends horizontal line at height of 1·25 m to the round tower, or some correct substitution into trig ratio / formula *Mid Partial Credit:* • 1 step correct *High Partial Credit:* • 2 steps correct • Correct equation in h, for example, $\sin 22 = \frac{h-1{\cdot}25}{25{\cdot}5}$ *Full Credit -1:* • Incorrect or no rounding

Q9	Model Solution – 50 Marks	Marking Notes

(a)
(i)
(ii)



Scale 15D (0, 4, 7, 10, 15)

7 values are needed in the tree diagram. Note that values may be correct, relative to values that are earlier in the diagram, or may be correct relative to 1.

Low Partial Credit:

- Work of merit, for example, finds 1 relevant probability

Mid Partial Credit:

- 3 values correct

High Partial Credit:

- 5 values correct

Full Credit –1:

- Incorrect rounding
- Apply a * if the probability that someone who has diabetes and gets a positive result is calculated as $\frac{0{\cdot}0663}{0{\cdot}067} = 0 \cdot 9896$

Q9	Model Solution – 50 Marks	Marking Notes
(a) **(iii)** **(iv)**	**(iii)** $0{\cdot}0663 + 0{\cdot}0728 = 0{\cdot}1391$ **(iv)** $\dfrac{0{\cdot}0663}{0{\cdot}1391} = 0 \cdot 47663$ $= 0 \cdot 4766\ [4\ \text{D.P.}]$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, in **(iii)**, one value correct; or in **(iv)**, top or bottom line correct • 1 part correct • Work of merit in both parts *High Partial Credit:* • 1 part correct **and** work of merit in the other
(b)	**Method 1** $P(\text{not having diabetes}) = 1 - 0{\cdot}067 = 0 \cdot 933$ $P(0) = \binom{5}{0}(0{\cdot}933)^5$ $= 0 \cdot 706981 \ldots$ $P(1) = \binom{5}{1}(0{\cdot}933)^4(0{\cdot}067)$ $= 0 \cdot 253846 \ldots$ $P(2\text{ or more}) = 1 - (0 \cdot 706981 \ldots + 0 \cdot 253846 \ldots)$ $= 1 - 0 \cdot 960828 \ldots$ $= 0 \cdot 03917 \ldots$ $= 0 \cdot 0392\ [4\ \text{D.P.}]$ **Method 2** $P(\text{not having diabetes}) = 1 - 0{\cdot}067 = 0 \cdot 933$ $P(2) = \binom{5}{2}(0{\cdot}067)^2(0{\cdot}933)^3$ $P(3) = \binom{5}{3}(0{\cdot}067)^3(0{\cdot}933)^2$ $P(4) = \binom{5}{4}(0{\cdot}067)^4(0{\cdot}933)$ $P(5) = \binom{5}{5}(0{\cdot}067)^5$ $P(2) + P(3) + P(4) + P(5) = 0 \cdot 0392\ [4\ \text{D.P.}]$	**Scale 5D (0, 2, 3, 4, 5)** For this part, "term" is taken to mean each of $P(0)$, $P(1)$, …, $P(5)$ *Low Partial Credit:* • Work of merit, for example, $P(2) + P(3) + P(4) + P(5)$, or $1 - 0{\cdot}067$, or $\binom{5}{0}$ *Mid Partial Credit:* • One term fully correct • One aspect of two terms correct (binomial coefficient, power of 0·933, power of 0·067) *High Partial Credit:* • Indicates how to find $P(2\text{ or more})$ **and** work indicated for *MPC* also present • Finds sufficient terms to find $P(2\text{ or more})$, but error(s) in finishing

Q9	Model Solution – 50 Marks	Marking Notes
(c) (i)	$\binom{20}{10} = 184\ 756$ **OR** $20_{P_{10}} \div 10!$ **OR** $\frac{20!}{10!\,10!}$	**Scale 5B (0, 2, 5)** Accept $\binom{20}{10}$, $20_{P_{10}} \div 10!$, or $\frac{20!}{10!10!}$ *Partial Credit:* • Work of merit, for example, finds $20_{P_{10}}$, $20!$ or $10!$
(c) (ii)	*Accept a solution that counts one combination of people in* **A**, *or a solution that counts all possible combinations for the people in* **A**. *One combination:* $10! = 3\ 628\ 800$ *All possible combinations:* $\binom{20}{10}(10!) = 184\ 756 \times 3\ 628\ 800$ $= 6{\cdot}7044 \ldots \times 10^{11}$	**Scale 5B (0, 2, 5)** Accept $10!$ or $\binom{20}{10}(10!)$ *Partial Credit:* • Work of merit, for example, $\binom{20}{10}$, or relevant diagram • $\binom{10}{1}$

Q9	Model Solution – 50 Marks	Marking Notes
(d)	*Accept a solution that counts one combination of people in* **X**, *or a solution that counts all possible combinations for the people in* **X**. *One combination:* **Method 1** $\binom{16}{2}\binom{14}{2}\binom{12}{2}\ldots\binom{2}{2} = 8{\cdot}1729\ldots\times 10^{10}$ $= 8{\cdot}173\times 10^{10}$ [3 D.P.] **Method 2** $\frac{16!}{2^8} = 8{\cdot}173\times 10^{10}$ [3 D.P.] **Method 3** $15\times 13\times 11\times\ldots\times 3\times 1\times(8!)$ $= 8{\cdot}173\times 10^{10}$ [3 D.P.] *All possible combinations:* **Method 1** $\binom{24}{8}\binom{16}{2}\binom{14}{2}\binom{12}{2}\ldots\binom{2}{2} = 6{\cdot}0109\ldots\times 10^{16}$ $= 6{\cdot}011\times 10^{16}$ [3 D.P.] **Method 2** $\binom{24}{8}\frac{16!}{2^8} = 6{\cdot}011\times 10^{16}$ [3 D.P.] **Method 3** $\binom{24}{8}\times 15\times 13\times 11\times\ldots\times 3\times 1\times(8!)$ $= 6{\cdot}011\times 10^{16}$ [3 D.P.]	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, $\binom{24}{8}$, or $\binom{24}{16}$, or $\binom{16}{2}$, or 8!, or diagram • $^{24}P_8$ or similar *High Partial Credit:* • Two relevant terms multiplied, for example, $\binom{24}{8}\binom{16}{2}$, or 15×13, or $\binom{16}{2}\binom{14}{2}$ • $\frac{16!}{2^8}$

Q10	Model Solution – 50 Marks	Marking Notes
(a)(i)	*Missing values:* $280, \;\; 340, \;\; 400, \;\; 460$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • 1 correct entry • Identifies μ or σ *Mid Partial Credit:* • 2 correct entries *High Partial Credit:* • 3 correct entries
(a)(ii)	$z = \dfrac{x-\mu}{\sigma}$ $z = \dfrac{420-400}{60}$ $= \dfrac{1}{3}$ $= 0{\cdot}33$ $1 - P(z < 0{\cdot}33) = 1 - 0{\cdot}6293$ $= 0.3707$ $= 0.37 \text{ [2 D.P.]}$ or accept: $1 - P(z < 0{\cdot}34) = 1 - 0{\cdot}6331$ $= 0.3669$ $= 0.37 \text{ [2 D.P.]}$	**Scale 10D (0, 3, 5, 7, 10)** Consider solution as consisting of 3 steps: **Step 1**. Finds z-score **Step 2**. Find $P(z < a)$ **Step 3**. Find required probability *Low Partial Credit:* • Work of merit, for example, normal curve drawn with some relevant work, or some correct substitution in $z = \dfrac{x-\mu}{\sigma}$ • Identifies μ or σ (if credit has not already been awarded in (i)) *Mid Partial Credit:* • 1 step correct *High Partial Credit:* • 2 steps correct

Q10	Model Solution – 50 Marks	Marking Notes
(b)	$\bar{x} \pm 1.96\dfrac{s}{\sqrt{n}}$ $387 \pm 1{\cdot}96\left(\dfrac{66{\cdot}2}{\sqrt{2161}}\right)$ $= 387 \pm 2{\cdot}79\ldots$ $384{\cdot}2 < \mu < 389{\cdot}8$ [1 D.P.]	**Scale 5D (0, 2, 3, 4, 5)** Two endpoints must be found for *FC*. Consider solution as consisting of 3 steps: **Step 1**. Finds $\frac{s}{\sqrt{n}}$ **Step 2**. Finds $1{\cdot}96 \times \frac{s}{\sqrt{n}}$ **Step 3**. $387 \pm 1{\cdot}96\left(\frac{66{\cdot}2}{\sqrt{2161}}\right)$ **Step 4**. Finds C.I. *Low Partial Credit:* • Work of merit, for example, $1{\cdot}96 \times s$, or any correct substitution in $\bar{x} \pm 1.96\dfrac{s}{\sqrt{n}}$ *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct • One endpoint of the interval found that is $\mu < 389.8$ or $\mu > 384.2$ *Full Credit -1:* • 387 ± 2.8
(c)	**(i)** $z = \dfrac{403 - 400}{\frac{70 \cdot 6}{\sqrt{2724}}}$ $= 2{\cdot}217\ldots$ $= 2 \cdot 22$ [2 D.P.] **(ii)** $\boldsymbol{p}$**-value**: $P(z < 2{\cdot}22) = 0{\cdot}9868$ $p = 2(1 - 0{\cdot}9868)$ $= 0.0264$ **Conclusion**: the mean score for country **Y** is significantly different to 400	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Work of merit, for example, in **(i)**, two numbers with a relevant operation, for example, $403 - 400$ or $\frac{70{\cdot}6}{\sqrt{2724}}$; or in **(ii)**, $P(z < 2{\cdot}22)$ indicated or found, or conclusion correct in **(ii)** • 1 part correct *High Partial Credit:* • 1 part correct **and** work of merit in the other part. *Full Credit -1:* • If the context is not mentioned in conclusion

Q10	Model Solution – 50 Marks	Marking Notes
(d)	**How to**: From all students who own a pet in country **Z**, take a random sample of $\frac{2520}{2} = 1260$. From all students who do not own a pet, do the same. **Why not useful**: Probably no connection between owning a pet and maths scores.	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, a general explanation of how to carry out stratified random sampling • 1 part correct *Full Credit -1:* • Random selection not mentioned, otherwise correct
(e)	For $\mathrm{E}[X]_{\max}$ let p be 0 $0{\cdot}19 + 2r + r = 1$ $3r = 0{\cdot}81$ $r = 0{\cdot}27$ $\mathrm{E}[X]_{\max} = 0(0{\cdot}19) + 1(0) + 2\big(2(0{\cdot}27)\big) + 3(0{\cdot}27)$ $= 1.89$	**Scale 15D (0, 4, 7, 10, 15)** Consider solution as consisting of 4 steps: **Step 1**. Sets $p = 0$ **Step 2**. Sets sum of probabilities $= 1$ **Step 3**. Finds r **Step 4**. Finds $E[X]$ *Low Partial Credit:* • Work of merit, for example, one relevant product from $\mathrm{E}[X]$ *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct