Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2024
Deferred Examinations

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the marking schemes for the deferred examinations

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. However, it should be noted that the marking schemes for the deferred examinations may not necessarily be as detailed as the corresponding marking schemes for the main sitting of an examination, which serve to ensure consistency across a large team of examiners.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination, and the need to maintain consistency in standards between the main sitting and the deferred sitting and from year to year. In the case of the deferred examinations, this means that the level of detail may vary by question, as the marking scheme will only have been finalised for the questions attempted by the candidates who sat these examinations.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with a senior examiner when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes (whether for the main examinations or the deferred examinations) should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination concerned. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination will not necessarily be the same for the deferred sitting as for the main sitting or from one year to the next.

Contents

Paper 1: Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5-mark scale	0, 5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10-mark scale		0, 4, 10	0, 4, 6, 10	0, 3, 5, 7, 10
15-mark scale			0, 6, 8, 15	0, 4, 6, 8, 15
20-mark scale				0, 8, 11, 13, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
*   response of no substantial merit (no credit)
*   correct response (full credit)

B-scales (three categories)
*   response of no substantial merit (no credit)
*   partially correct response (partial credit)
*   correct response (full credit)

C-scales (four categories)
*   response of no substantial merit (no credit)
*   response with some merit (low partial credit)
*   almost correct response (high partial credit)
*   correct response (full credit)

D-scales (five categories)
*   response of no substantial merit (no credit)
*   response with some merit (low partial credit)
*   response about half-right (mid partial credit)
*   almost correct response (high partial credit)
*   correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded.

The only marks that may be awarded for a question are those on the scale above, or *Full Credit -1*.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

In general, an answer without sufficient supporting work is awarded the lowest non-zero level of credit (typically *Partial Credit* or *Low Partial Credit*, as appropriate).

Paper 1: Summary of mark allocations and scales to be applied

Section A (150) Answer any five questions				Section B (150) Answer any three questions			

Section A (150)
Answer any five questions

Question 1 (30)
(a)	10C
(b)	10C
(c)	10D

Question 2 (30)
(a)(i)	10C
(a)(ii)	10D
(b)	10D

Question 3 (30)
(a)	10C
(b)	10C
(c)	10D

Question 4 (30)
(a)	10C
(b)	10C
(c)	10D

Question 5 (30)
(a)	10D
(b)(i)	5B
(b)(ii)	15D

Question 6 (30)
(a)(i)	5B
(a)(ii)	15D
(b)(i),(ii)	10D

Section B (150)
Answer any three questions

Question 7 (50)
(a)(i)(ii)	10D
(b)	20D
(c)	10C
(d)	10C

Question 8 (50)
(a)	5C
(b)	10D
(c)	10C
(d)	5C
(e)	10C
(f)	10C

Question 9 (50)
(a)	5B
(b)	10D
(c)	10C
(d)	10D
(e)(i)	5B
(f)(ii)	10D

Question 10 (50)
(a)	5C
(b)	10C
(c)	5C
(d)(i)	10C
(d)(ii)	10C
(e)	10D

Paper 1: Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	$5(x^2 - 9)$ $5(x + 3)(x - 3)$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, a common factor identified; mentions difference of 2 squares *High Partial Credit* • $(x + 3)(x - 3)$
(b)	$6xy - 8x + 21y - 28$ $2x(3y - 4) + 7(3y - 4)$ $(3y - 4)(2x + 7)$ **OR** $6xy - 8x + 21y - 28$ $3y(2x + 7) - 4(2x + 7)$ $(3y - 4)(2x + 7)$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, a common factor identified *High Partial Credit* • $2x(3y - 4) + 7(3y - 4)$ • $3y(2x + 7) - 4(2x + 7)$
(c)	$2y = -7 - 4x$ $y = \dfrac{-7 - 4x}{2}$ $x^2 + 4x\left(\dfrac{-7 - 4x}{2}\right) = -168$ $x^2 - 14x - 8x^2 = -168$ $x^2 + 2x - 24 = 0$ $(x - 4)(x + 6) = 0$ $x = 4 \ \ or \ \ x = -6$ $y = \dfrac{-7 - 4(4)}{2} \qquad y = \dfrac{-7 - 4(-6)}{2}$ $y = \dfrac{17}{2} \qquad\qquad y = -\dfrac{23}{2}$ $x = 4, \ y = \dfrac{17}{2} \ \ or \ \ x = -6, \ y = \dfrac{-23}{2}$	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit* • Work of merit, for example, effort to isolate x or y *Mid Partial Credit* • Quadratic in one variable *High Partial Credit* • Finds correct values for x (or y)

Q2	Model Solution – 30 Marks	Marking Notes
(a) **(i)**	$a = 3, \qquad d = 7$ $S_n = \dfrac{n}{2}[2a + (n-1)d]$ $S_{100} = \dfrac{100}{2}[2(3) + (100-1)7]$ $S_{100} = 34950$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, identifies a or d *High Partial Credit* • Fully correct substitution into S_n formula
(a) **(ii)**	$T_4 = 3 + 3(7) = 24,$ $r = 8$ $T_3 = 8 \times 24$ $\quad = 192$ $3, 24, 192, \ldots$ $T_n = a + (n-1)d$ $192 = 3 + (n-1)7$ $189 = (n-1)7$ $27 = n - 1$ $n = 28$	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit* • Work of merit, for example, calculates T_4 or identifies the correct common ratio *Mid Partial Credit* • Calculates 192 correctly • Finds T_n for the arithmetic sequence *High Partial Credit* • Sets up the correct equation in n
(b)	To Prove: $P(n) = 1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$ $P(1): 1 = \dfrac{1(1+1)}{2} = 1$, True Assume $P(k)$ is true ie. $P(k) = 1 + 2 + 3 + \cdots + k = \dfrac{k(k+1)}{2}$ To Prove: $P(k+1) = \dfrac{(k+1)(k+2)}{2}$ $P(k+1) = 1 + 2 + 3 + \cdots + k + (k+1)$ $\quad = \dfrac{k(k+1)}{2} + (k+1)$ $\quad = \dfrac{k(k+1)}{2} + \dfrac{2(k+1)}{2}$ $\quad = \dfrac{(k+1)(k+2)}{2}$ But $P(1)$ is true, so $P(2)$ is true etc. Hence, $P(n)$ is true for all n.	**Scale 10D (0, 3, 5, 7, 10j)** *Low Partial Credit* • Any one of Step $P(1)$, Step $P(k)$ or Step $P(k+1)$ *Mid Partial Credit* • Any two of Step $P(1)$, Step $P(k)$ or Step $P(k+1)$ *High Partial Credit* • $P(1), P(k)$ and some valid work in using Step $P(k)$ to prove Step $P(k+1)$ *Full Credit-1* • Omits conclusion but otherwise correct

Q3	Model Solution – 30 Marks	Marking Notes
(a)	$g(6x - 5)$ $(6x - 5)^2 - 2(6x - 5) + 1$ $36x^2 - 72x + 36$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Some correct substitution of $f(x)$ into $g(x)$ • $f(g(x))$ substituted *High Partial Credit* • $f(x)$ fully substituted for x in $g(x)$ • $f(g(x))$ in the form $ax^2 + bx + c$
(b)	$$\frac{1}{3^4} = 3^{-4}$$ **OR** $$\frac{1}{81} = 3^r$$ $$r = \log_3 \frac{1}{81}$$ $$r = -4$$ Ans: 3^{-4}	**Scale 10C (0, 4, 6, 10)** *Partial Credit* • Work of merit, for example, lets $r = \log_3 \frac{1}{81}$, writes 81 as 3^4 or $\frac{1}{81}$ as 81^{-1} *High Partial* • $\frac{1}{3^4}$ *Full Credit -1* • $r = -4$
(c)	$\log_{100} x + \log_{100} 18 - \log_{100} 3$ $= \log_{100} \dfrac{18x}{3}$ $= \log_{100} 6x$ $= \dfrac{\log_{10} 6x}{\log_{10} 100}$ $= \dfrac{\log_{10} 6x}{2}$ $= \dfrac{1}{2} \log_{10} 6x$	**Scale 10D (0, 3, 5, 7, 10)** **3 steps:** **1.** Combines all 3 terms into 1 term **2.** Changes to the base 10 **3.** Finishes *Low Partial Credit* • Work of merit, for example, some correct use of rules of logs *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct

Q4	Model Solution – 30 Marks	Marking Notes
(a)	$6 - 8i + 15i - 20i^2 - 20i + 2i^2$ $= 6 - 8i + 15i + 20 - 20i - 2$ $= 24 - 13i$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, some correct multiplication *High Partial Credit* • Expands correctly [1st line in solution]
(b)	Correctly labelled diagram	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, one dot labelled correctly, or z and $-z$ swapped. *High Partial Credit* • 2 dots labelled correctly
(c)	$r = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = 2$ $\tan A = \frac{\sqrt{3}}{1}$, so $A = \frac{\pi}{3}$, so $\theta = \frac{5\pi}{3}$ $z^2 = 1 - \sqrt{3}i$ $z^2 = 2\left(\cos\left(\frac{5\pi}{3} + 2n\pi\right) + i\sin\left(\frac{5\pi}{3} + 2n\pi\right)\right)$ $z = 2^{\frac{1}{2}}\left(\cos\left(\frac{5\pi}{6} + n\pi\right) + i\sin\left(\frac{5\pi}{6} + n\pi\right)\right)$ **$n = 0$:** $z = \sqrt{2}\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$ $z = \sqrt{2}\left(\left(-\frac{\sqrt{3}}{2}\right) + i\left(\frac{1}{2}\right)\right) = -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$ **$n = 1$:** $z = \sqrt{2}\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right)$ $z = \sqrt{2}\left(\left(\frac{\sqrt{3}}{2}\right) + i\left(-\frac{1}{2}\right)\right) = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$	**Scale 10D (0, 3, 5, 7, 10)** Note: polar form must be used to achieve any credit Note that argument may also be given as $\theta = -60°$, etc., or $\theta = 300°$, etc. **4 steps:** 1. Finds r 2. Finds θ 3. Uses de Moivre's Theorem 4. Evaluates *Low Partial Credit* • Work of merit, for example, plots $1 - \sqrt{3}i$, or some correct substitution into de Moivre's Theorem *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q5	Model Solution – 30 Marks	Marking Notes
(a)	$f(x) = \frac{6x^2}{2} + 4x + c$ $f(x) = 3x^2 + 4x + c$ $f(-1) = 3(-1)^2 + 4(-1) + c$ $8 = 3 - 4 + c$ $c = 9$ $f(x) = 3x^2 + 4x + 9$	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit* • Some correct integration *Mid Partial Credit* • Two terms correct in integration *High Partial Credit* • $3(-1)^2 + 4(-1) + c = 8$ • Finds $f(-1)$ in terms of c *Full Credit -1* • c found correctly
(b) (i)	$y = 2\ln 2^3 = 4.1589$	**Scale 5B (0, 2, 5)** • Some correct substitution into the function
(b) (ii)	$u = x \qquad\qquad v = \ln x^3$ $\dfrac{du}{dx} = 1 \qquad \dfrac{dv}{dx} = \dfrac{1}{x^3}(3x^2) = \dfrac{3}{x}$ $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ $\dfrac{dy}{dx} = x\left(\dfrac{3}{x}\right) + \ln x^3 \,(1)$ $\dfrac{dy}{dx} = 3 + \ln x^3$ $\dfrac{dy}{dx} = 3 + 3\ln x$	**Scale 15D (0, 4, 6, 8, 15)** Note: No differentiation no credit **4 steps:** **1.** Finds $\dfrac{du}{dx}$ **2.** Finds $\dfrac{dv}{dx}$ **3.** Substitutes correctly in the product rule **4.** Finishes *Low Partial Credit* • Some correct differentiation *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q6	Model Solution – 30 Marks	Marking Notes
(a) **(i)**	$h'(x) = 2\cos 2x$	**Scale 5B (0, 2, 5)** • Some correct differentiation
(a) **(ii)**	$\dfrac{1}{\dfrac{\pi}{6} - 0} \displaystyle\int_0^{\frac{\pi}{6}} \sin 2x \, dx$ $= \dfrac{6}{\pi}\left[-\dfrac{\cos 2x}{2}\right]_{x=0}^{x=\frac{\pi}{6}}$ $= \dfrac{6}{\pi}\left(\dfrac{-\cos 2(\frac{\pi}{6})}{2}\right) - \dfrac{6}{\pi}\left(\dfrac{-\cos 2(0)}{2}\right)$ $= \dfrac{6}{\pi}\left(-\dfrac{1}{4} + \dfrac{1}{2}\right)$ $= \dfrac{3}{2\pi}$	**Scale 15D (0, 4, 6, 8, 15)** Note: integration is required in order to be awarded any credit **4 steps** (if $\frac{1}{\frac{\pi}{6}}$ is omitted, treat step 1 as not fully correct, but all other steps can be accepted as correct): 1. $\frac{1}{\frac{\pi}{6}}\int_0^{\frac{\pi}{6}} \sin 2x \, dx$ 2. Integrates correctly 3. Subs in limits 4. Evaluates correctly *Low Partial Credit* • Work of merit, for example, integration indicated *Mid Partial Credit* • 2 steps correct *High 2 Partial Credit* • 3 steps correct
(b) **(i)** **(ii)**	(i) $1 < x < 2$ (ii) $a < 3$	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit:* • Work of merit *Mid Partial Credit:* • Work of merit in both parts • One part correct *High Partial Credit:* • One part correct and work of merit in the other

Q7	Model Solution – 50 Marks	Marking Notes
(a)	(i)	**Scale 10D (0, 3, 5, 7, 10)**

(a) (i)

0	5	10	15	20	25
-900	-925	**2200**	**2625**	1900	**-275**

(ii) Graph

Scale 10D (0, 3, 5, 7, 10)

Solution consists of 11 parts:

    -4 values in table

    -6 points plotted

    -Points joined appropriately

Low Partial Credit
- 1 part correct

Mid Partial Credit
- 5 parts correct

High Partial Credit
- 8 parts correct

Full Credit -1
- 10 parts correct

(b)

(i)

$g(-16)=$
$$-2(-16)^3 + 55(-16)^2 + 974(-16) - 6688$$
$$= 0$$

(ii)

$$\begin{array}{r} -2x^2 + 87x - 418 \\ \hline x+16 \,\big|\, -2x^3 + 55x^2 + 974x - 6688 \\ -2x^3 - 32x^2 \\ \hline 87x^2 + 974x \\ 87x^2 + 1392x \\ \hline -418x - 6688 \\ -418x - 6688 \end{array}$$

$2x^2 - 87x + 418 = 0$

$(2x - 11)(x - 38) = 0$

$$x = \frac{11}{2} \text{ and } x = 38$$

Scale 20D (0, 8, 11, 13, 20)

4 steps:
1. Shows $g(-16) = 0$
2. Finds quadratic factor
3. Factorises $-2x^2 + 87x - 418$
OR fully substituted formula
4. Finds other roots

Low Partial Credit
- Work of merit, for example, some correct division, or sets up long division correctly

Mid Partial Credit
- 1 step correct and work of merit in another

High Partial Credit
- 3 steps correct

Q7	Model Solution – 50 Marks	Marking Notes
(c)	$(21)(-12)(-47) = 11844$ $-1316 \div 11844 = -\dfrac{1}{9}$ $h(x) = -\dfrac{1}{9}(x + 21)(x - 12)(x - 47)$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, some correct factor identified *High Partial Credit* • $h(x) = (x + 21)(x - 12)(x - 47)$ • Finds $a = -\dfrac{1}{9}$
(d)	<u>Standard Tax:</u> $40\,000@20\% = 0.2(40000)$ <u>Higher Tax:</u> $(x - 40\,000)@40\% = 0.4(x - 40000)$ <u>Net Tax:</u> $0.2(40000) + 0.4(x - 40000) - 3550$ <u>Equation:</u> $0.2(40000) + 0.4(x - 40000) - 3550 = \dfrac{x}{6}$ $8000 + 0.4x - 16000 - 3550 = \dfrac{x}{6}$ $\dfrac{x}{6} = 0.4x - 11550$ $x = 2.4x - 69300$ $1.4x = 69300$ $x = [€]49,500$	**Scale 10C (0, 3, 5, 7, 10)** Consider solution as consisting of 4 steps **1.** Finds tax @20% **2.** Finds tax @40% **3.** Deals with tax credit **4.** Finds x *Low Partial Credit* • Work of merit, for example, calculates 8000 or writes $\dfrac{x}{6}$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$w(0) = 5 + 20e^{-0.079(0)} = 25[\%]$ $w(8) = 5 + 20e^{-0.079(8)} = 15.63\% \approx 16[\%]$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example some correct substitution into the function *High Partial Credit* • One correct value calculated *Full Credit-1* • No rounding or incorrect rounding
(b)	$5 + 20e^{-0.079t} \le 10$ $20e^{-0.079t} \le 5$ $e^{-0.079t} \le \dfrac{1}{4}$ $-0.079t \le \ln\dfrac{1}{4}$ $t \ge \dfrac{\ln\dfrac{1}{4}}{-0.079}$ $t \ge 17.54802989$ [weeks] $t \ge 122.8362092$ [days] $t = 123$ [days]	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit:* • Work of merit, for example, trials values of t *Mid Partial Credit:* • Isolates $e^{-0.079t}$ *High Partial Credit:* • Isolates t *Full Credit -1* • Rounding the number of weeks and then calculating the number of days
(c)	$w(t) = 5 + 20e^{-0.079t}$ $w'(t) = 20e^{-0.079t}(-0.079)$ $w'(5) = 20e^{-0.079(5)}(-0.079)$ $w'(t) = -1.06[\%$ per week$]$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, some correct differentiation *High Partial Credit* • Differentiates correctly
(d)	The water content of the wood is always decreasing. The rate at which it is decreasing by each week is also decreasing.	**Scale 5C (0, 2, 5)** *Partial Credit* • Explains the meaning of $w'(t)$ being always negative • Explains the meaning of $|w'(t)|$ getting smaller as t gets bigger

Q8	Model Solution – 50 Marks	Marking Notes
(e)	**(i)** Graph under the given graph starting and approaching the same limit of 5. **(ii)** k is any value less than -0.079	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit *High Partial Credit* • 1 part correct *Full Credit -1* • $y = u(t) < w(t)$ for all t but not approaching 5.
(f)	Year 1: $1.06P$ Year 2: $1.08 \times 1.06P = 1.1448P$ Year 3: $1.1448P \times 1.17 = 1.339416P$ $$(1 + i)^3 = 1.339416$$ $$1 + i = \sqrt[3]{1.339416}$$ $$i = 1.1023 - 1$$ $$= 0.1023$$ $$r = 10.2$$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, writes 1.06 *High Partial Credit* • Calculates 1.339416 • Finds total % increase over the 3 years

Q9	Model Solution – 50 Marks	Marking Notes
(a)	$g(1.5) = -6(1.5)^2 + 25(1.5) - 24 = 0$	**Scale 5B (0, 2, 5)** *Partial Credit* • Work of merit, for example, correct substitution into given expression or attempt to factorise quadratic • $-6x^2 + 25x - 24 = 0$
(b)	$f'(x) = -\dfrac{3}{4}x^2 + 2x$ $-\dfrac{3}{4}x^2 + 2x = 0$ $3x^2 - 8x = 0$ $x(3x - 8) = 0$ $x = 0$ and $x = \dfrac{8}{3}$ $f\left(\dfrac{8}{3}\right) = -\dfrac{\left(\frac{8}{3}\right)^3}{4} + \left(\dfrac{8}{3}\right)^2 = \dfrac{64}{27}$ Local Maximum: $\left(\dfrac{8}{3}, \dfrac{64}{27}\right)$	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit* • Work of merit, for example, some correct differentiation; states $f'(x) = 0$ *Mid Partial Credit* • Fully correct differentiation *High Partial Credit* • Finds $x = \dfrac{8}{3}$
(c)	$f'(x) = -\dfrac{3}{4}x^2 + 2x$ $f''(x) = -\dfrac{6}{4}x + 2$ $-\dfrac{6}{4}x + 2 = 0$ $\dfrac{6}{4}x = 2$ $x = \dfrac{4}{3}$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example mentions $f''(x)$ • Brings down derivative from (b) *High Partial Credit* • Finds $f''(x)$

Q9	Model Solution – 50 Marks	Marking Notes

(d)	$\int_0^2 (f(x))dx$
	
	$= \int_0^2 \left(-\frac{x^3}{4} + x^2\right) dx$
	
	$= \left(-\frac{x^4}{16} + \frac{x^3}{3}\right)_{x=0}^{x=2}$
	
	$\left(-\frac{2^4}{16} + \frac{2^3}{3}\right) - \left(-\frac{0^4}{16} + \frac{0^3}{3}\right) = \left(\frac{5}{3} - 0\right) = \frac{5}{3}$
	
	$\int_{1.5}^2 (g(x))dx$
	
	$= \int_0^2 (-6x^2 + 25x - 24)dx$
	
	$= \left(-2x^3 + \frac{25}{2}x^2 - 24x\right)_{x=0}^{x=2}$
	
	$= \left(-2(2)^3 + \frac{25}{2}(2)^2 - 24(2)\right) -$
	$\qquad \left(-2(1.5)^3 + \frac{25}{2}(1.5)^2 - 24(1.5)\right)$
	
	$= (-14) - \left(-\frac{117}{8}\right)$
	
	$= \frac{5}{8}$
	
	Area $= \frac{5}{3} - \frac{5}{8} = \frac{25}{24}$ [units^2]
		Scale 10D (0, 4, 6, 10)
		
		Consider 3 Steps:
		  1. Integrate f
		  2. Integrate g
		  3. Evaluate with limits and finish
		
		Low Partial Credit:
		  • Work of merit, for example, integration indicated
		
		High Partial Credit:
		  • 2 steps correct
		  • One relevant area calculated

(e) (i)	$V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi r^2 (2r)$ $V = \frac{2}{3}\pi r^3$	**Scale 5B (0, 2, 5)** *Partial Credit* • Lets $h = 2r$

Q9	Model Solution – 50 Marks	Marking Notes
(e) (ii)	$V = \frac{2}{3}\pi r^3$ $\frac{dV}{dr} = 2\pi r^2$ $\frac{dr}{dV} = \frac{1}{2\pi r^2}$ $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ $\frac{dr}{dt} = \left(\frac{1}{2\pi r^2}\right) \times (0.5)$ $$\frac{dr}{dt} = \left(\frac{1}{4\pi r^2}\right)$$ $$= \frac{1}{4\pi(4)^2}$$ $$= \frac{1}{64\pi} \text{ cm/s}$$	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit* • Work of merit, for example, states a relevant derivative or some correct differentiation *Mid Partial Credit* • Any two of the following: ○ $\frac{dV}{dt} = 0.5$ ○ $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ ○ $\frac{dV}{dr} = 2\pi r^2$ *High Partial Credit* • Have all 3 from MPC list *Full Credit -1* • Incorrect or no units

Q10	Model Solution – 50 Marks	Marking Notes
(a)	<table><tr><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>**120**</td><td>96</td><td>$\frac{384}{5}$</td><td>$\frac{1536}{25}$</td></tr></table>	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit *High Partial Credit* • 2 entries correct *Full Credit-1* • Answer(s) in decimal form
(b)	$a = 120$ and $r = 0.8$ $T_n = ar^{n-1}$ $10 = (120)(0.8)^{n-1}$ $\frac{1}{12} = (0.8)^{n-1}$ $\log_{0.8}\frac{1}{12} = n - 1$ $11.1359 = n - 1$ $12.1359 = n$ $p = 13$	**Scale 10C (0, 4, 6, 10)** Note: Solve by trial and improvement T_{12} and T_{13} must be evaluated for full credit *Low Partial Credit* • Work of merit, for example, identifies a or r • Isolates 0.8^{n-1} • Trials some values *High Partial Credit* • Converts to a log equation • $p = 13$ with T_{13} evaluated
(c)	$S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{120(1-0.8^n)}{1-0.8}$ $[S_n = 600(1 - 0.8^n)]$	**Scale 5C (0, 2, 5)** *Partial Credit* • Work of merit, for example, some correct substitution into the S_n formula
(d) (i)	$|AC| = |AB| - |CD|$ $|AC| = 120 - 96 = 24$cm $|AD| = |AC| + |CD|$ $|AD| = 24 + 76.8 = 100.8$cm	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit, for example, $|AC| = |AB| - |CD|$ *High Partial Credit* • One distance calculated *Full Credit -1* • Incorrect unit or no unit

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(d) **(ii)**	$\|AX\| = \|AB\| - \|BC\| + \|CD\| - \cdots.$ $\|AX\| = 120 - 96 + \dfrac{384}{5} - \cdots$ $a = 120$ and $r = -0.8$ $S_\infty = \dfrac{a}{1-r}$ $S_\infty = \dfrac{120}{1-(-0.8)}$ $S_\infty = \dfrac{200}{3} \, \text{cm}$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit* • Work of merit *High Partial Credit* • Correct substitution into S_∞ formula *Full Credit -1* • Incorrect unit or no unit
(e)	*Pipe A:* Fills $\dfrac{5}{18}$ of the container in 5 minutes *Pipe B:* Fills $\dfrac{5}{15}$ of the container in 5 minutes <u>Total fraction filled after 5 minutes:</u> $\dfrac{5}{18} + \dfrac{5}{15} = \dfrac{11}{18}$ <u>Fraction left to fill:</u> $1 - \dfrac{11}{18} = \dfrac{7}{18}$ <u>Time to fill:</u> $\dfrac{7}{18} \div \dfrac{1}{15} + 5 = \dfrac{65}{6}$ $= 10$ minutes 50 seconds	**Scale 10D (0, 3, 5, 7, 10)** *Low Partial Credit* • Work of merit, for example, states $\dfrac{1}{18}$ or similar *Mid Partial Credit* • Calculates $\dfrac{11}{18}$, the fraction filled by both pipes in five minutes *High Partial Credit* • Calculates $\dfrac{7}{18}$, the fraction left to fill after 5 minutes • Answer left as $\dfrac{65}{6}$