2023.M30

2023L003A2EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2023

Mathematics

Paper 2

Higher Level

Monday 12 June   Morning 9:30 - 12:00

300 marks

Examination Number

Day and Month of Birth

For example, 3rd February is entered as 0302

Centre Stamp

The 2023 examination papers were adjusted to compensate for disruptions to learning due to COVID-19. This examination paper does not necessarily reflect the same structure and format as the examination papers of past or subsequent years.

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer **any five** questions from Section A.

Answer **any three** questions from Section B.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

In general, diagrams are not to scale.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

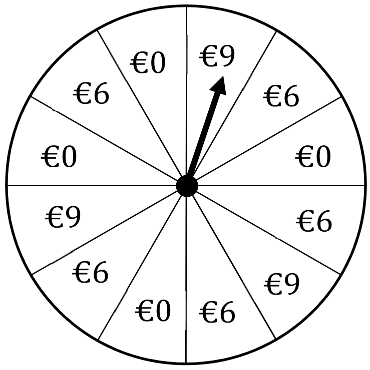
Section A Concepts and Skills 150 marks

Answer **any five** questions from this section.

Question 1 **(30 marks)**

A circular spinner has 12 sectors, as follows:

- 5 sectors are labelled €6
- 3 sectors are labelled €9
- The rest are labelled €0.



In a game, the spinner is spun once.
The spinner is equally likely to land on each sector.
The player gets the amount of money shown on the sector that the spinner lands on.

(a) Fiona plays the game a number of times.

Work out the probability that Fiona gets €6, then €9, then €6 the first three times she plays.
Give your answer correct to 4 decimal places.

(b) Rohan also plays the game a number of times.

Find the probability that Rohan gets €9 for the 3rd time, on the 8th time that he plays the game. Give your answer correct to 4 decimal places.

(c) Olga plays the game 2 times.

Find the probability that Olga gets less than €16 in **total** from playing the game.
Give your answer correct to 4 decimal places.

Question 2 **(30 marks)**

(a) Prove that $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

(b) Using the formula in **part (a)**, and without using a calculator, find the value of $\sin 75°$.
Give your answer in surd form.

(c) Find all solutions of the following equation in t, for $0° \leq t \leq 360°$:

$$\sin t = \sin(2t)$$

Question 3 **(30 marks)**

(a) Find the area of the triangle with vertices $(4, 6)$, $(-3, -1)$, and $(0, 11)$.

(b) $A(-1, k)$ and $B(5, l)$ are two points, where $k, l \in \mathbb{Q}$.

(i) Show that the midpoint of $[AB]$ is $\left(2, \frac{k+l}{2}\right)$.

(ii) The perpendicular bisector of $[AB]$ is:

$$3x + 2y - 14 = 0$$

Find the value of l and the value of k.

$l =$ ____________________ $k =$ ____________________

Question 4 **(30 marks)**

(a) The circle c has equation:

$$(x-h)^2+(y+3)^2=12$$

where $h \in \mathbb{R}$.

(i) Write down the centre and radius of the circle c.
Give your answer in terms of h, where appropriate.

Centre = (,   )   Radius = ____________

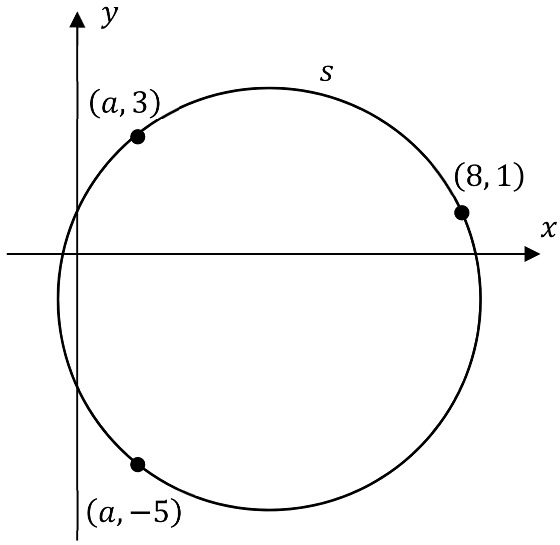
(ii) The perpendicular distance from the line $x-4y+7=0$ to the centre of the circle c is 5 units.

Work out the two possible values of h. Give each answer in surd form.

(b) The circle s passes through the points $(8, 1)$, $(a, 3)$, and $(a, -5)$, as shown in the diagram on the right (not to scale), where $0 < a < 5$, $a \in \mathbb{R}$.

The radius of the circle s is $\sqrt{20}$.

Find the equation of the circle s.



Question 5 **(30 marks)**

Rohan has a large number of small cubes. The cubes are identical in size.

(a) Some of the cubes are red, some are green, and the rest are blue.
Rohan carries out an experiment in which he picks out 5 different cubes at random.
He records the number of cubes of each colour, and replaces the 5 cubes.
He then repeats this process a number of times.

The table below shows the number of cubes of each colour the first 7 times Rohan does this, labelled Trial A to Trial G.

Trial	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Number of red cubes	0	3	2	2	4	5	1
Number of green cubes	4	2	0	3	0	0	2
Number of blue cubes	1	0	3	0	1	0	2

(i) Work out the mean and standard deviation of the number of **red** cubes per trial, for these 7 trials. Give each answer correct to 1 decimal place.

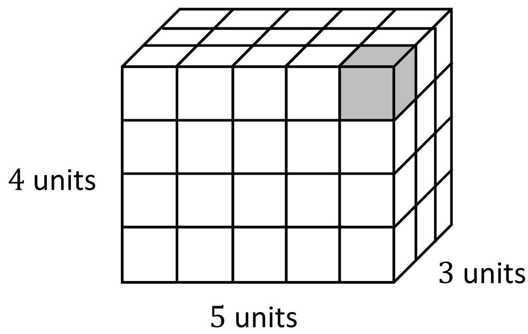
(ii) Work out the correlation coefficient between the number of **red** cubes and the number of **green** cubes per trial, for these 7 trials.
Give your answer correct to 3 decimal places.

Answer, $r =$

(iii) Rohan repeats this experiment a large number of times.
Explain why you would expect the correlation coefficient between the number of red cubes and the number of green cubes per trial to be **negative**.

(b) Rohan makes a solid cuboid of dimensions $5 \times 3 \times 4$ using his small cubes.
Each small cube has sides of length 1 unit.
Some of the small cubes have 1 face, 2 faces, or 3 faces on the outside of the cuboid.
Other small cubes have no faces on the outside of the cuboid.

In the diagram below, the small cube that is shaded has 3 faces on the outside of the cuboid.



Fill in the table below, showing the number of small cubes with 3 faces, 2 faces, 1 face, or no faces on the outside of this cuboid. Show your working out.
One of the values is filled in for you.

Number of small cubes with 3 faces on the outside of the cuboid:	
Number of small cubes with 2 faces on the outside of the cuboid:	
Number of small cubes with 1 face on the outside of the cuboid:	22
Number of small cubes with no faces on the outside of the cuboid:	

Question 6 **(30 marks)**

(a) State whether the following statement is true or false:

Two angles are vertically opposite if, and only if, they are equal in size.

Justify your answer.

Answer (true or false):

Justification:

(b) The two diagrams below show the same rectangle, $ACEG$ (not to scale).
The points B, D, and F lie on $[AC]$, $[CE]$, and $[EG]$, respectively, as shown.
$|AB| = 20$ cm, $|BC| = 30$ cm, and $|AG| = 90$ cm.
$|\angle GFA| = |\angle EFD| = |\angle DBC| = \theta$, where $\theta \in \mathbb{R}$.

In **Diagram B** below, $[GE]$ and $[BD]$ are extended, and they meet at the point H.

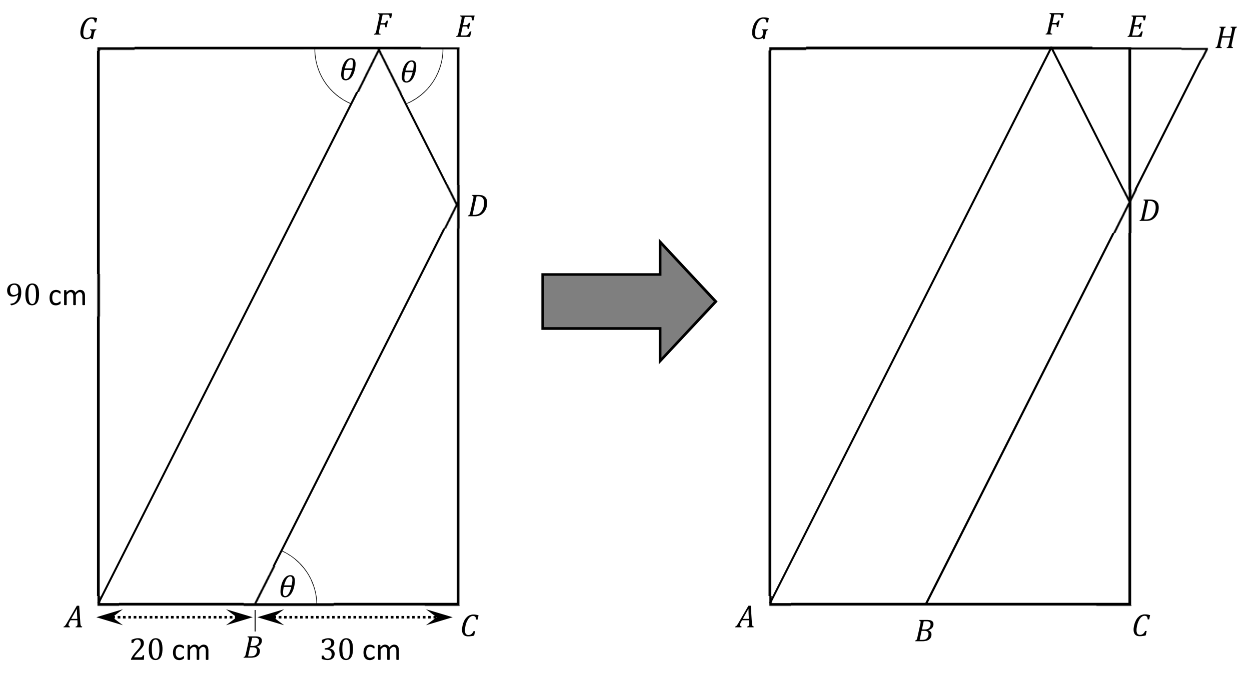


Diagram A **Diagram B**

(i) Prove that $|FE| = |EH|$, in **Diagram B**. Use congruent triangles.
Give a reason for each statement that you make in your proof.

(ii) Hence, or otherwise, find the size of the angle θ.
Give your answer correct to the nearest degree.

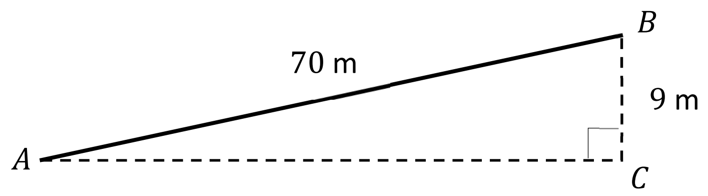
Section B — Contexts and Applications — 150 marks

Answer **any three** questions from this section.

Question 7 **(50 marks)**

Olga is a cyclist.

(a) The diagram below shows a road $[AB]$, which is not to scale.
AC is horizontal and BC is vertical.
$|BC| = 9$ m and $|AB| = 70$ m.

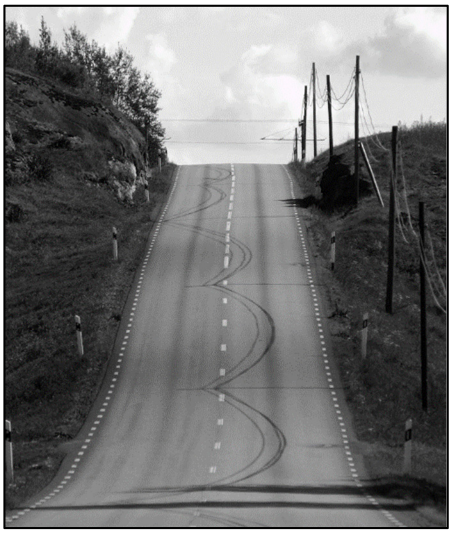


The gradient of the road $[AB]$ is $\frac{|BC|}{|AC|}$ written as a **percentage**.

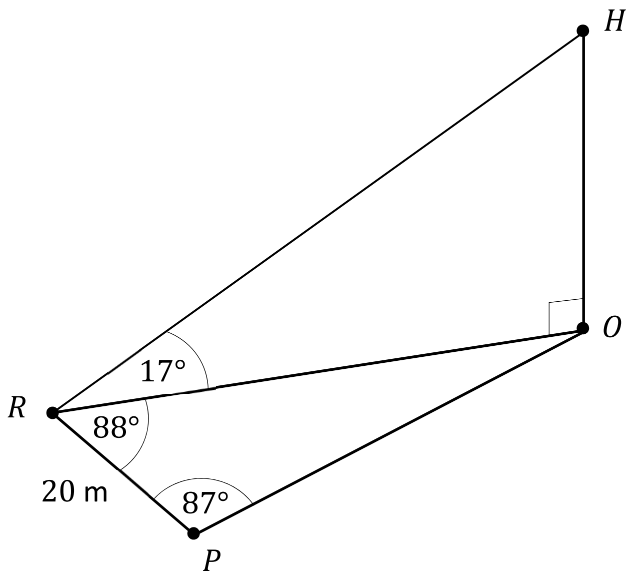
Find the gradient of $[AB]$, correct to the nearest percent.

(b) Olga wants to measure the vertical height of a hill. The point H is at the top of the hill. The points R and P are 20 m apart on horizontal ground, at the bottom of the hill.

Olga measures the angle of elevation from R to H.
Taking O to be the point directly below H that is horizontal with R and P, Olga also measures the angles $\angle OPR$ and $\angle ORP$.
All of these are shown in the diagram below (not to scale).



Source: www.bikeforums.net/road-cycling



Work out the distance $|OH|$, the vertical height of the top of the hill relative to the points R and P. Give your answer correct to the nearest metre.

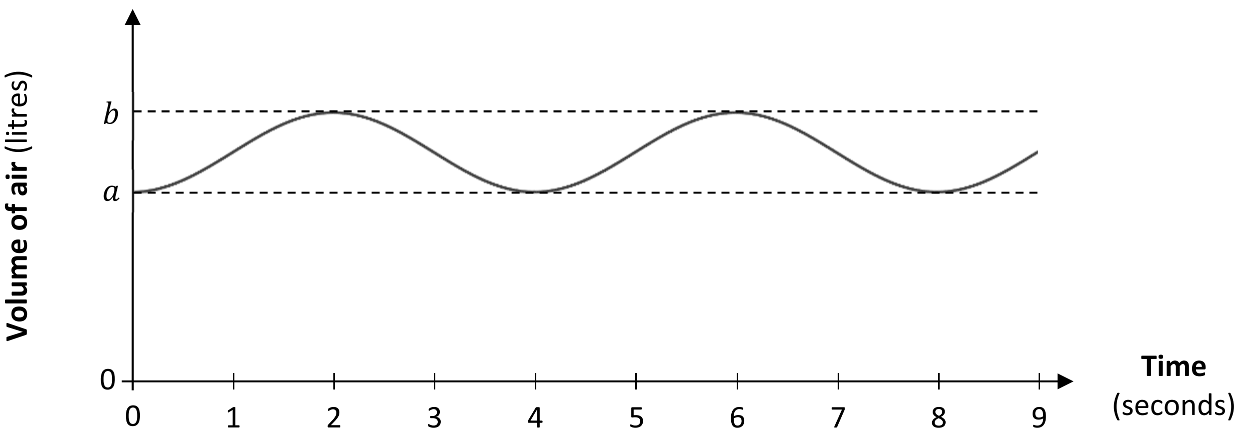
This question continues on the next page.

Olga has some tests done to measure her lung capacity.
When she is resting, the volume of air, V, in her lungs after t seconds can be modelled by:

$$V(t) = 2 - 0{\cdot}4\cos\left(\frac{\pi}{2}t\right),$$

where V is in litres, $t \geq 0$ is the time in seconds from a given point in time, and $\frac{\pi}{2}t$ is in **radians**.

The diagram below shows the graph of the function $y = V(t)$ for the first 9 seconds.



(c) Find the values marked a and b on the graph, the minimum and maximum values of V.

(d) What is the connection between $V'(t)$, the derivative of V, and whether Olga is breathing in or breathing out?

(e) Use the formula $V(t) = 2 - 0{\cdot}4\cos\left(\frac{\pi}{2}t\right)$ to find each of the following, when Olga is resting. Give each answer correct to 3 decimal places.

(i) Find the volume of air in Olga's lungs, half a second after $t = 0$.

(ii) Find the rate at which the volume of air in Olga's lungs is increasing, half a second after $t = 0$.

(f) Olga's breathing is also measured while she is doing gentle exercise. During this time:

- when she breathes in fully, the volume of air in her lungs is $3{\cdot}6$ litres
- when she breathes out fully, the volume of air in her lungs is $1{\cdot}3$ litres
- she breathes in and out **twice** as many times per minute as when she is resting.

Use this information to write a formula for $E(t)$, the volume of air in Olga's lungs during this time, t seconds after she has breathed out fully. Give your answer in the form $E(t) = a + b\cos(ct)$, where $a, b, c, t \in \mathbb{R}$, and ct is in radians.

$E(t) =$

Question 8 **(50 marks)**

An online word game involves trying to guess a five-letter word in as few attempts as possible. For this game, each player is given a score s, where $s \in \mathbb{R}$, based on how many attempts it takes them to guess the word.

(a) In Ireland, players' scores are approximately normally distributed, with a mean of $3{\cdot}87$ and a standard deviation of $0{\cdot}36$.

A player is selected at random from the players in Ireland.
Find the probability that this player has a score of less than $3{\cdot}5$.

(b) A random sample of 64 Galway players has a mean score of $3{\cdot}74$. Based on this, a local newspaper claims that Galway players have a different mean score to players in Ireland.

(i) Use the information about this sample to construct a 95% confidence interval for the mean score of **all** Galway players. Use the standard deviation of $0{\cdot}36$ in your calculations.

(ii) Carry out a hypothesis test at the 5% level of significance to test the newspaper's claim that Galway players have a different mean score to players in Ireland.

State your null hypothesis, state your alternative hypothesis, state your conclusion, and give a reason for your conclusion.

Null Hypothesis:

Alternative Hypothesis:

Conclusion:

Reason for your conclusion:

(c) A national newspaper conducts a survey on a random sample of n teenagers in Ireland. 35% of the sample said they play the online word game every day.

Based on this, a 95% confidence interval for the percentage, p, of all teenagers in Ireland who play the game every day was calculated, as accurately as possible.
Correct to one decimal place, this interval was: $26{\cdot}5\% \leq p \leq 43{\cdot}5\%$.

Use this to work out the value of n, the number of teenagers surveyed.

This question continues on the next page.

(d) A player wins the game if they guess the word with 6 guesses or less.
The organisers of the game record the following statistics for players in Europe:

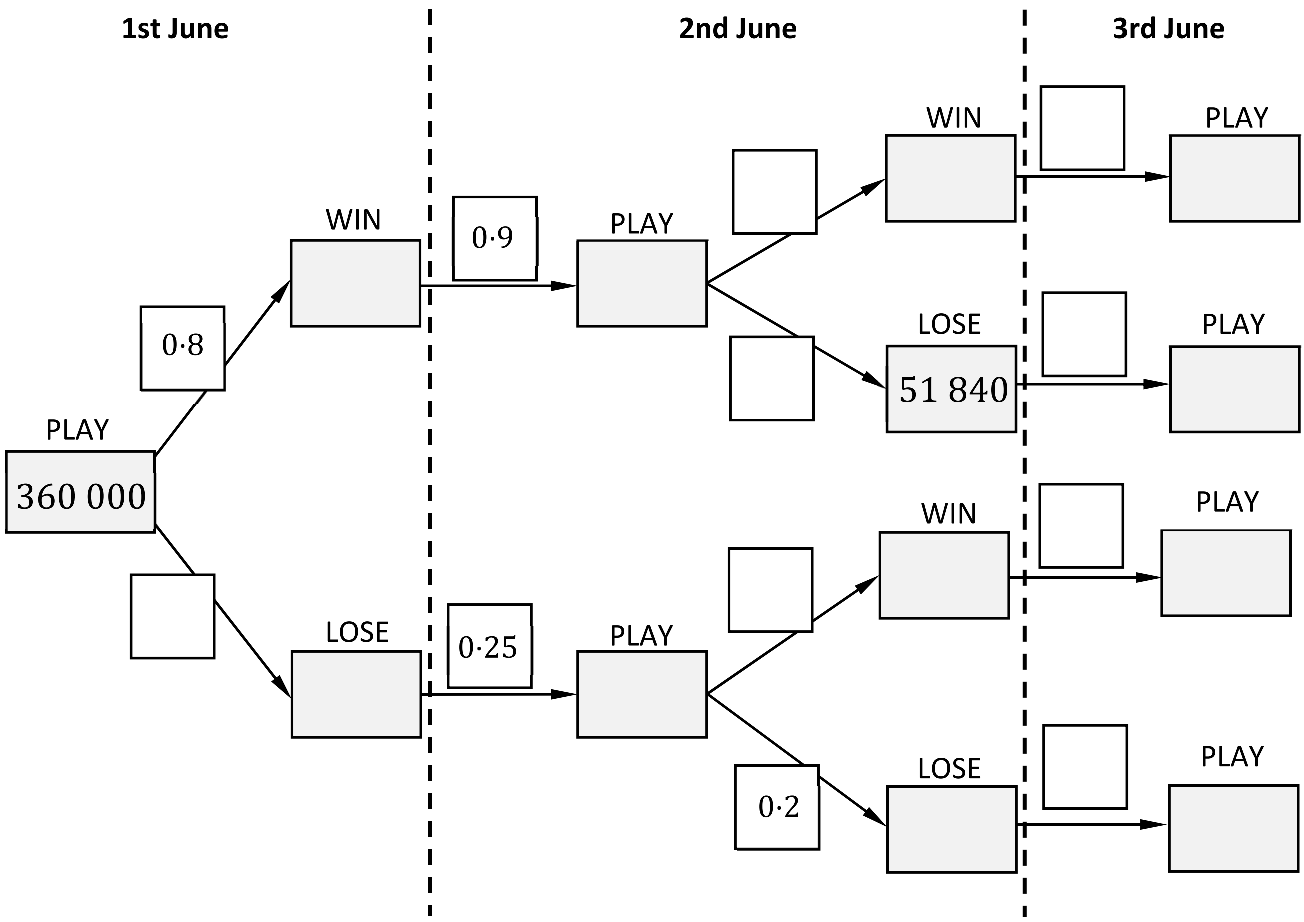
- 80% of players win the game each day
- 90% of players who win the game will play it again the following day
- 25% of players who do **not** win the game will play it again the following day.

Some of this information is shown in the tree diagram below, based on the $360\ 000$ people who played the game on 1st June. The squares represent proportions. The rectangles labelled PLAY, WIN, and LOSE represent the number of people who play, win, and lose the game, respectively. For example, $0{\cdot}8$ of the $360\ 000$ people who played the game on 1st June won the game.

Assume that, for people who play the game on a number of days, winning the game on one day is independent of winning it on another day.

(i) Complete the tree diagram, by writing the proportion associated with each branch of the tree diagram into the appropriate square **and** writing the number of people who play, win, and lose the game into the appropriate rectangle.

There is space for working out on the next page.



Space for working out.

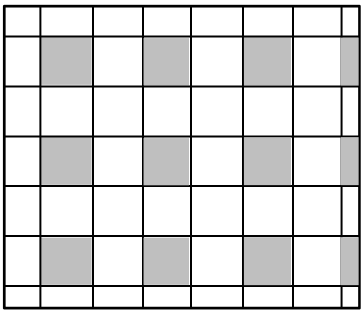
(ii) One person is picked at random from those who played the game on all three days (1st, 2nd, and 3rd June).

Find the probability that this person lost on 1st June or 2nd June (or on both).

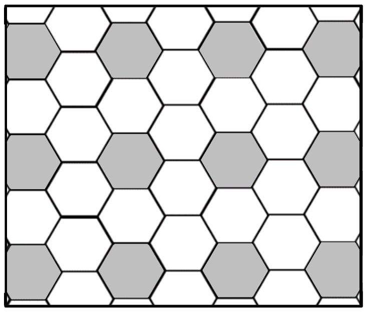
Question 9 **(50 marks)**

Ava is looking at different tilings, that is, different ways of covering a region with shapes.

(a) First, she looks at two different tilings: one using identical squares and another using identical regular hexagons.
(*Note*: a regular hexagon can be split into 6 congruent equilateral triangles.)



Tiling with squares

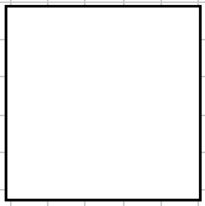


Tiling with regular hexagons

A square tile and a hexagonal tile each have an **area** of 140 cm^2.

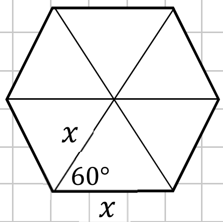
(i) Work out the length of the side of the square tile, correct to 1 decimal place.

Area of square: 140 cm^2

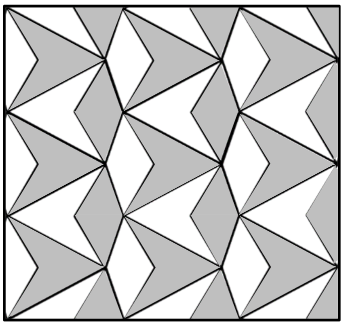


(ii) The hexagonal tile has sides of length x cm, where $x \in \mathbb{R}$.
Work out the value of x, correct to 1 decimal place.

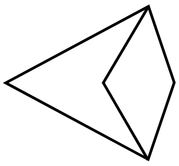
Area of hexagon: 140 cm^2

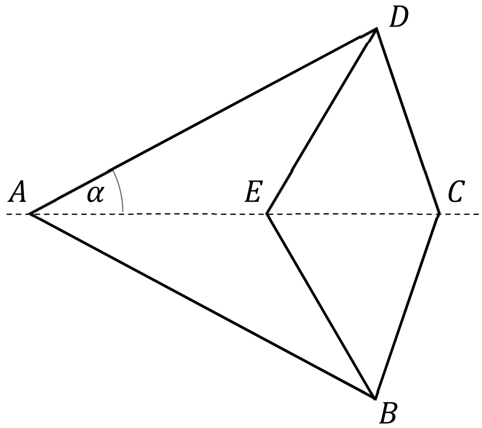


(b) Next, Ava looks at more complicated tilings. The tiling below is made up of two shapes: an arrowhead ($ABED$) and a quadrilateral ($EBCD$). The point E lies on the line AC, and both shapes are **symmetrical** about AC (diagrams not to scale).



Tiling





(i) $|AD| = 8$ cm, $|AE| = 6$ cm, and $|ED| = 4$ cm.
As shown in the diagram, $\alpha = \angle DAE$. Show that $\alpha = \cos^{-1}\left(\frac{7}{8}\right)$.

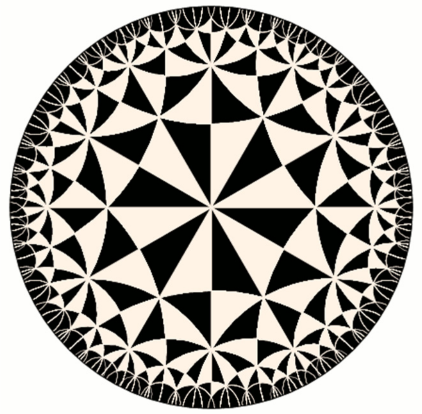
(ii) In the diagram $|\angle ADC| = |\angle ABC| = 90°$.

Use this, and part **(b)(i)**, to work out the total area of the quadrilateral $ABCD$, correct to 2 decimal places.

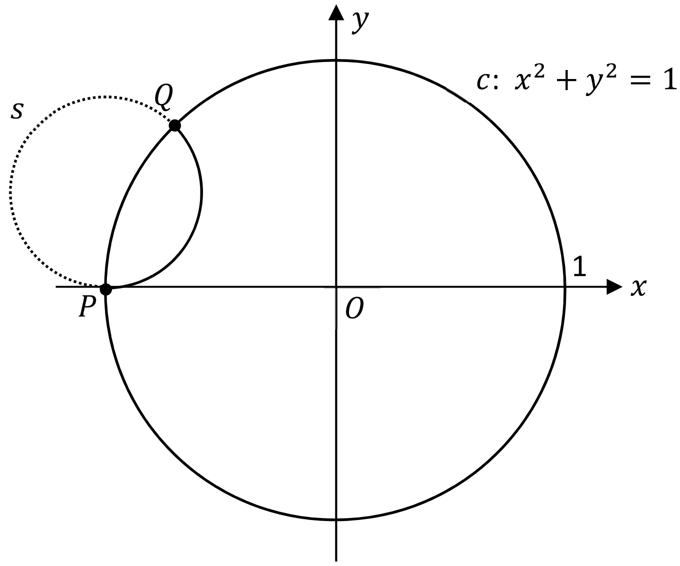
This question continues on the next page.

(c) Ava also looks at a tiling of the inside of the unit circle c: $x^2 + y^2 = 1$.
The tiling she looks at is shown on the left below.

The diagram on the right below (not to scale) shows the circle c and another circle, s.
The points P and Q are on both circles.
The part of s that lies inside c is an edge of a number of tiles.
Ava wants to find the equation of the circle s.



Tiling of inside of circle



(i) $|\angle QOP| = 45°$, where O is the point $(0, 0)$.

Show that the point Q has co-ordinates $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

Image on left above: Coxeter, H.S.M., cited in
https://web.colby.edu/thegeometricviewpoint/2016/12/21/tessellations-of-the-hyperbolic-plane-and-m-c-escher/

(ii) The point P lies on the x-axis.

The centre of the circle s lies on the tangent to c at the point P **and** on the tangent to c at the point Q.

Find the centre and the radius of the circle s. Give your answers in surd form.

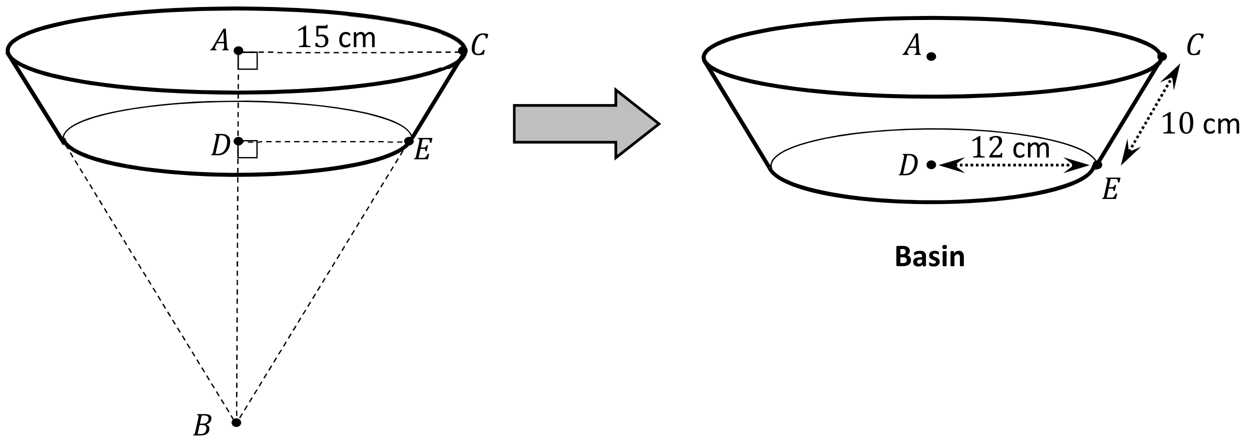
Centre $= (\quad , \quad)$

Radius $=$ __________

Question 10 **(50 marks)**

(a) Séamus has a basin in the shape of an inverted right circular cone with the lower part removed, as shown in the diagram on the right below (diagrams not to scale).

The radius of the original cone is 15 cm, and $|CE|$, the slant height of the basin, is 10 cm. The base of the basin is a horizontal circle with a radius of 12 cm, as shown. The basin is open – it does not have a top.



(i) Show that $|BE| = 40$ cm.

(ii) Find the **total** internal surface area of the basin, correct to 1 decimal place.

(iii) Draw a diagram of the net of the **curved** surface of the basin.
Show your working out.

Include on your diagram enough **measurements** of lengths and/or angles so that the net could be constructed using a ruler, straight edge, compass, and protractor, without any further calculation.

Your diagram may need to include lines that are not part of the net itself, like the broken lines in the diagram on the previous page.

This question continues on the next page.

(b) Séamus needs to pick a new PIN code. It must be a 4-digit code.
It must use exactly 4 of the digits from 1 to 9, so no digit can be used more than once.

(i) Work out how many such codes are possible.

(ii) Work out how many of these codes contain the digit 2.

(iii) Work out how many of the codes in **part (b)(i)** have the **sum** of their first three digits equal to their fourth digit. (The codes can contain the digit 2, although they do not have to.)

Page for extra work.
Label any extra work clearly with the question number and part.

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Leaving Certificate – Higher Level

Mathematics Paper 2

Monday 12 June

Morning 9:30 - 12:00