Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2020

Marking Scheme

Mathematics

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 3, 4, 5	0, 2, 3, 4, 5	
10 mark scales	0, 10	0, 5, 10	0, 4, 8, 10	0, 3, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 5, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

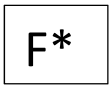
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.
Throughout the scheme indicate by use of ***** where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A		***Section B***	
Question 1	(25 marks)	Question 7	(35 marks)
(a)(i)	10C	(a)(i)	10C
(a)(ii)	5D	(a)(ii)	5C
(b)	10D	(b)	5C
		(c)(i)	5C
Question 2	(25 marks)	(c)(ii)	10C
(a)	10C		
(b)	15D	Question 8	(55 marks)
		(a)	5B
Question 3	(25 marks)	(b)(i)	5C
(a)(i)(ii)	10D	(b)(ii)	10C
(a)(iii)	5C	(c)(i)	5C
(b)	10C	(c)(ii)	5C
		(c)(iii)	5C
Question 4	(25 marks)	(d)(i)	10C
(a)(i)	10C	(d)(ii)	10D
(a)(ii)	5C		
(b))	10C	Question 9	(60 marks)
		(a)	10C
Question 5	(25 marks)	(b)(i)	10B
(a)	5C	(b)(ii)	5C
(b)(i)	5C	(b)(iii)	5C
(b)(ii)	10C	(c)(i)	10C
(b)(iii)	5C	(c)(ii)	5C
		(d)(i)	10C
Question 6	(25 marks)	(d)(ii)	5C
(a)(i)	10C		
(a)(ii)	5C		
(a)(iii)	5C		
(b)	5D		

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
	Tick	Work of relevance	The work presented in the body of the script merits full credit
	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
	Star	Rounding or Unit or Arithmetic error Misreading	
	Horizontal wavy	Error	
	Tick L		The work presented in the body of the script merits low partial credit
	Tick M		The work presented in the body of the script merits mid partial credit (or partial credit)
	Tick H		The work presented in the body of the script merits high partial credit
	F star		The work presented in the body of the script merits Full Credit – 1
	Left Bracket		Another version of this solution is presented elsewhere and is merits equal or higher credit
	Vertical wavy	No work on this page (portion of the page)	
	Oversimplify	The candidate has oversimplified the work	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

e.g. In a **C scale** where * and and appear in the body of the work then should be placed in the right margin.

In the case of a **D scale** with the same level of annotation then should be placed in the right margin.

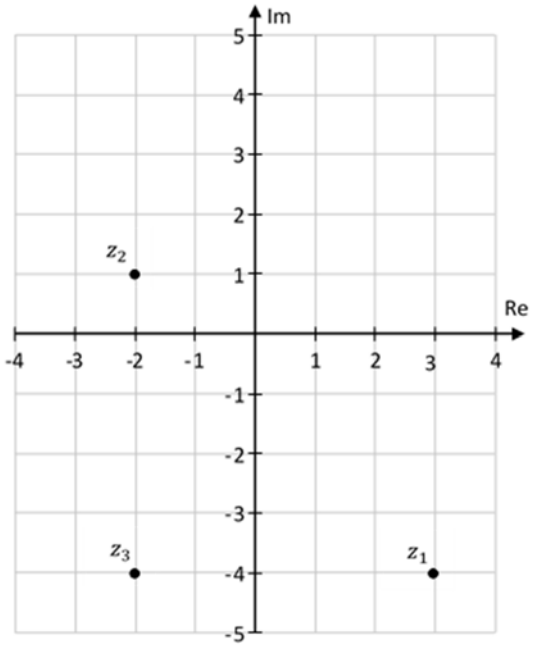
A in the body of the work may sometimes be used to indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is then indicated in the right margin.

Model Solutions & Detailed Marking Notes

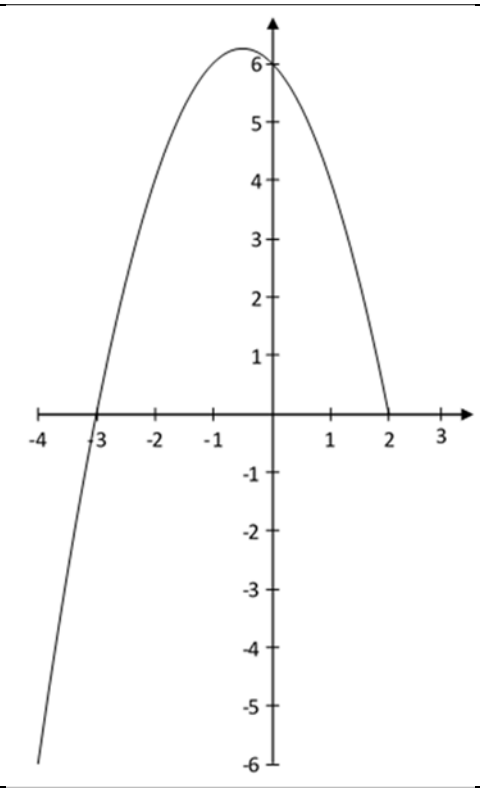
Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

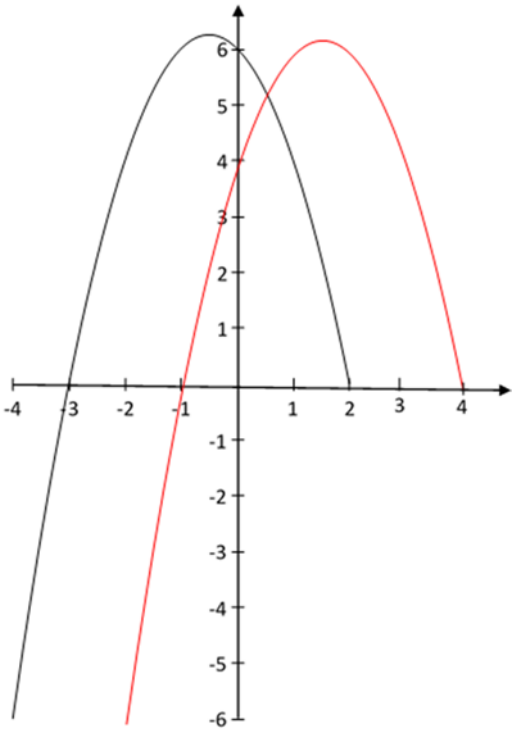
Q1	Model Solution – 25 Marks	Marking Notes
(a) (i)	$45 \times 12{\cdot}60 + (350 - 200) \times 0{\cdot}22$ $= €600$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $45 \times 12{\cdot}60$ $(350) \times 0{\cdot}22$ $(200) \times 0{\cdot}22$ *High Partial Credit:* $45 \times 12{\cdot}60 + (350 - 200) \times 0{\cdot}22$ Either pay found **Note**: Accept correct answer without work **Note**: $45 \times 12{\cdot}60 + (350) \times 0{\cdot}22$ finished correctly merits HPC
(a) (ii)	$48 \times 12{\cdot}60 + 3(12{\cdot}6 \times 1{\cdot}5) = 661{\cdot}50$ $(713{\cdot}20 - 661{\cdot}50) \div 0{\cdot}22 = €235{\cdot}00$ $€235{\cdot}00 + 200 = €435{\cdot}00$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* $48 \times 12{\cdot}60$ $3(12{\cdot}6)$ *Mid Partial Credit:* Commission found or €661·50 *High Partial Credit:* $(713{\cdot}20 - 661{\cdot}50) \div 0{\cdot}22$
(b)	$678 \times 0{\cdot}20 + (713{\cdot}20 - 678) \times 0{\cdot}40$ $= €149{\cdot}68$ $€149{\cdot}68 - 26 = 123{\cdot}68$ $€713{\cdot}20 - 123{\cdot}68$ $= €589{\cdot}52$ Net Income	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $678 \times 0{\cdot}20$ $713{\cdot}20 - 678$ *Mid Partial Credit:* Gross tax found *High Partial Credit:* Net tax found

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$12\left(\frac{9x-6}{2}\right)=12\left(\frac{3x-14}{3}\right)+12\left(\frac{9x}{4}\right)$ $6(9x-6)=4(3x-14)+3(9x)$ $54x-36=12x-56+27x$ $x=\frac{-20}{15}$ or $-\frac{4}{3}$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* 12 identified as common denominator Some relevant multiplication $4(3x-14)=3(9x)$ without CD *High Partial Credit:* $54x-36=12x-56+27x$ or equivalent
(b)	$y=3x-4$ $4x^2-3x(3x-4)=4$ $4x^2-9x^2+12x-4=0$ $-5x^2+12x-4=0$ $5x^2-12x+4=0$ $(5x-2)(x-2)=0$ $x=\frac{2}{5}$ and $x=2$ $y=3\left(\frac{2}{5}\right)-4=\frac{-14}{5}$ $y=3(2)-4=2$ $\left(\frac{2}{5},-\frac{14}{5}\right)$ and $(2,2)$	**Scale 15D (0, 5, 7, 11,15)** *Low Partial Credit:* x or y isolated *Mid Partial Credit:* $4x^2-3x(3x-4)=4$ or equivalent equation in one variable *High Partial Credit:* 1 relevant value (root) found from quadratic

Q3	Model Solution – 25 Marks	Marking Notes
(a) (i) + (a) (ii)	$z_3 = 2i(-2+i) = -4i + 2i^2$ $-2 - 4i$  	**Scale10D (0, 3, 5, 8, 10)** **Note**: 4 items (find z_3 and 3 plots) *Low Partial Credit:* One correct item (correct plot for incorrect z_3) or (z_1 plotted correctly) or (z_2 plotted correctly) or (z_3 correct but with incorrect plot) *Mid Partial Credit:* 2 correct items *High Partial Credit:* 3 correct items
(a) (iii)	$\lvert z_1 \rvert = \sqrt{9+16} = 5$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $\lvert z_1 \rvert$ formula *High Partial Credit:* Formula fully substituted
(b)	$z_4 = \dfrac{(29+3i)(3+4i)}{(3-4i)(3+4i)}$ $= \dfrac{87+116i+9i+12i^2}{9-16i^2}$ $= \dfrac{75+125i}{25}$ $= 3+5i$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Some correct substitution Conjugate identified Some multiplication above and below by same number, even if incorrect conjugate *High Partial Credit:* $\dfrac{(29+3i)(3+4i)}{(3-4i)(3+4i)}$

Q4	Model Solution – 25 Marks	Marking Notes
(a) (i)	$\frac{10\,000\,000}{3{\cdot}28^2} = 929506{\cdot}2463$ $= 930\,000$ m^2 $9{\cdot}3 \times 10^5$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $10\,000\,000 \div 3{\cdot}28$ or equivalent $10\,000\,000 \times 3{\cdot}28^2$ *High Partial Credit:* $10\,000\,000 \div 3{\cdot}28^2$
(a) (ii)	$\frac{930\,000}{5}$ $= 186\,000$ litres $186\,000 \div 25$ $= 7440$ tins	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Relevant use of answer (a)(i) *High Partial Credit:* Incorrect litres ÷ 25 correctly
(b)	$2^{9x-1} = (2^3)^{2x}$ $2^{9x-1} = 2^{6x}$ $\Rightarrow \ 9x - 1 = 6x$ $\Rightarrow \ x = \frac{1}{3}$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* 2^3 or 2^{6x} List of powers of 2 *High Partial Credit:* Equation in x (Indices handled correctly)

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$(x+1)(x-4)=0$ $x=-1 \quad \text{or} \quad x=4$ Or Formula $x=\frac{3\pm\sqrt{25}}{2}$ $x=-1 \quad \text{or} \quad x=4$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Correct roots formula *a* or *b* or *c* explicitly identified Effort at factorising Correct answer without work *High Partial Credit:* Correct factors 1 correct root $x=\frac{3\pm\sqrt{25}}{2}$ and stops
(b) **(i)**	x | -4 | -3 | -2 | -1 | 0 | 1 | 2 $f(x)$ | −6 | 0 | 4 | 6 | 6 | 4 | 0	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* One correct table entry *High Partial Credit:* Three correct table entries
(b) **(ii)**		**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* One correct plot from table entries *High Partial Credit:* Three correct plots from table entries with some join

(b) **(iii)**		**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* One correct point or table entry or plot for $g(x)$ *High Partial Credit:* Three correct points or table entries or plots for $g(x)$

Q6	Model Solution – 25 Marks	Marking Notes
(a) (i)	$f'(x) = 12x^2 - 6x + 1$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* 1 term [other than −7 (see note)] correctly differentiated *High Partial Credit:* 2 terms correctly differentiated **Note**: differentiation of independent term must be explicit for marks to be awarded i.e. $f'(-7) = 0$
(a) (ii)	$12(1)^2 - 6(1) + 1 = 7$ (Slope)	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Any substitution into candidates $f'(x)$ $f'(1)$ or equivalent written *High Partial Credit:* $f'(-1)$ fully substituted
(a) (iii)	$y + 5 = 7(x - 1)$ $7x - y - 12 = 0$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Eq. of line formula with some substitution *High Partial Credit:* Eq. of line formula fully substituted with $(1, 5)$ but not candidates answer from (a)(ii)
(b)	$g(2) = 2(2)^2 + p(2) + q$ $8 + 2p + q = 6$ $2p + q = -2$ $g'(3) = 4(3) + p = 9$ $\Rightarrow p = -3$ Sub $p = -3$ $\Rightarrow 2(-3) + q = -2$ $q = 4$ $p = -3 \qquad q = 4$	**Scale5D (0, 2, 3, 4, 5)** *Low Partial Credit:* $g'(x)$ found $g(2)$ found *Mid Partial Credit:* $g'(x)$ and $g(2)$ found $2p + q = -2$ *High Partial Credit:* p found

Section B

Q7	Model Solution – 35 Marks	Marking Notes
(a) (i)	$443{\cdot}66 \times 12 \times 3 = €15\,971{\cdot}76$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Some relevant substitution into formula *High Partial Credit:* Formula fully substituted
(a) (ii)	$\frac{15971{\cdot}76}{12\,000} \times 100 = 133{\cdot}09 = 133{\cdot}1\%$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Some relevant substitution into formula *High Partial Credit:* Formula fully substituted
(b)	$(1+r)^3 = 1{\cdot}331$ $1 + r = 1{\cdot}1$ $r = 0{\cdot}1 = 10\%$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Some relevant substitution into formula *High Partial Credit:* Formula fully substituted
(c) (i)	$32\,000(0{\cdot}8)^3 = €16\,384$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Some relevant substitution into formula *High Partial Credit:* Formula fully substituted
(c) (ii)	$x(0{\cdot}8)^3 = 17920$ $x = \frac{17920}{0{\cdot}8^3} = €35\,000$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Some relevant substitution into formula *High Partial Credit:* Formula fully substituted (x isolated)

Q8	Model Solution – 55 Marks	Marking Notes
(a)	$h(0) = \frac{3}{5}$ m	**Scale 5B (0, 2, 5)** *Partial Credit:* $h(0)$ indicated and stops
(b) **(i)**	$\frac{1}{60}(12)^2 - \frac{1}{4}(12) + \frac{3}{5} = 0$ $0 = 0$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Some relevant substitution into formula *High Partial Credit:* Formula fully substituted
(b) **(ii)**	$60[\frac{1}{60}x^2 - \frac{1}{4}x + \frac{3}{5} = 0]$ $x^2 - 15x + 36 = 0$ $(x-3)(x-12) = 0$ $x = 3$ m	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $h(x) = 0$ *High Partial Credit:* Function factorised correctly Division of function by $(x-3)$ indicated
(c) **(i)**	$h'(x) = \frac{1}{30}x - \frac{1}{4}$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* 1 term [other than $\frac{3}{5}$ (see note)] correctly differentiated *High Partial Credit:* 2 terms correctly differentiated **Note**: differentiation of independent term must be explicit for marks to be awarded i.e. $f'\left(\frac{3}{5}\right) = 0$

(c) (ii)	$\frac{1}{30}x - \frac{1}{4} = 0$ $x = \frac{30}{4} = \frac{15}{2}$ m	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $f'(x) = 0$ explicit *High Partial Credit:* $\frac{1}{30}x - \frac{1}{4} = 0$
(c) (iii)	$h(7{\cdot}5) = \frac{1}{60}(7{\cdot}5)^2 - \frac{1}{4}(7{\cdot}5) + \frac{3}{5}$ $= \frac{27}{80}$ m	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Some substitution into $h(x)$ *High Partial Credit:* $h(x)$ fully substituted with candidates value from (c)(ii)

(d) (i)	$\frac{29{\cdot}33 - 28{\cdot}5}{28{\cdot}5} \times 100 = 2{\cdot}9\%$ $\frac{30{\cdot}68 - 29{\cdot}33}{29{\cdot}33} \times 100 = 4{\cdot}6\%$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Relevant use of question data to formulate a percentage *High Partial Credit:* One correct percentage found
(d) (ii)	$\frac{x - 25{\cdot}01}{25{\cdot}01} \times 100 = 14{\cdot}7$ $\Rightarrow \ x = 28{\cdot}68$ $\frac{y - 28{\cdot}68}{28{\cdot}68} \times 100 = 2{\cdot}91$ $\Rightarrow \ y = 29{\cdot}51$ $\frac{z - 29{\cdot}51}{29{\cdot}51} \times 100 = 4{\cdot}6$ $\Rightarrow \ z = 30{\cdot}87$ Total $= 25{\cdot}01 + 28{\cdot}68 + 29{\cdot}51 + 30{\cdot}87$ $=$1:54·07 Difference $=$ 1:54·07 – 1:53·36 $=0{\cdot}71$ Seconds	**Scale10D (0, 3, 5, 8, 10)** *Low Partial Credit:* 50 – 100 m split time formulated using 25·01 and $14{\cdot}7\%$ *Mid Partial Credit:* One correct split time found *High Partial Credit:* Three correct split times found

Race Section	Split Time (secs)	Percentage Increase
0 – 50 m	25·01	
50 – 100 m	28·68	$14{\cdot}7\%$
100 – 150 m	29·51	$2{\cdot}9\%$
150 – 200 m	30·87	$4{\cdot}6\%$
Total	$1{:}54{:}07$	

Q9	Model Solution – 60 Marks	Marking Notes
(a)	3, 7, **11, 15, 19, 23**	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* One correct or consistent table entry *High Partial Credit:* Three correct or consistent table entries
(b) (i)	39	**Scale 10B (0, 5, 10)** *Partial Credit:* Evidence of counting beyond T_6 Note: Accept correct answer without work
(b) (ii)	$T_n = 3 + (n-1)4$ $T_n = 4n - 1$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* T_n formula written *High Partial Credit:* Formula with some substitution
(b) (iii)	$4n - 1 = 147$ $n = 37$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* T_n formula written *High Partial Credit:* Formula fully substituted

(c) (i)	$S_n = \frac{n}{2}[6 + (n-1)4]$ $= \frac{n}{2}[4n + 2]$ $S_n = 2n^2 + n$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* S_n formula written *High Partial Credit:* Formula with some substitution
(c) (ii)	$2n^2 + n = 820$ $2n^2 + n - 820 = 0$ $(2n + 41)(n - 20) = 0$ $n = 20$ patterns	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* S_n formula written $S_n = 820$ Evidence of counting of sums *High Partial Credit:* Quadratic equation factorised **Note**: Accept correct answer without work
(d) (i)	$1, 3, \mathbf{5}, \mathbf{7}, \mathbf{9}, \mathbf{11}$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* One correct or consistent table entry *High Partial Credit:* Three correct or consistent table entries
(d) (ii)	$4\sqrt{3} + 3(4\sqrt{3}) + 5(4\sqrt{3}) + 7(4\sqrt{3}) + \cdots$ $= 4\sqrt{3}(1 + 3 + 5 + 7 + \cdots)$ $= 4\sqrt{3}(S_{15})$ where $a = 1$ and $d = 2$ $= 4\sqrt{3}\left[\frac{15}{2}(2 + 14 \times 2)\right]$ $= 4\sqrt{3}[7{\cdot}5 \times 30]$ $= 1558{\cdot}845727 = 1559$ cm²	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* At least 3 terms of AP identified S_n formula written *High Partial Credit:* Formula fully substituted