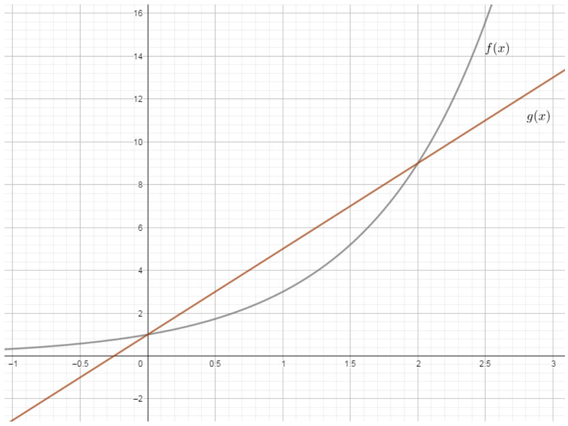
Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$(2x+1)(x^2+px+4)$ $2x^3+2px^2+8x+x^2+px+4$ $8+p=2(2p+1)$ $8+p=4p+2$ $3p=6$ $p=2$ Or Coefficient of x is $8+p$ Coefficient of x^2 is $2p+1$ $8+p=2(2p+1)$ $8+p=4p+2$ $3p=6$ $p=2$	**Scale10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant multiplication *Mid Partial credit:* - Multiplication completed without error(**s**) - Multiplication completed with errors and correctly identifies (in terms of p) the coefficient of **either** x^2 or x - Correctly identifies the coefficient of **either** x or x^2 *High Partial credit:* - Multiplication completed with error(s) but finishes correctly without further errors - Relevant coefficients equated (equation in p) - Multiplication completed and coefficients of x^2 and x identified but solves incorrect equation in p

(b)	$$\frac{3}{2x+1}+\frac{2}{5}=\frac{2}{3x-1}$$ CD: $5(2x+1)(3x-1)$ $$15(3x-1)+(4x+2)(3x-1)$$ $$=10(2x+1)$$ $$12x^2+27x-27=0$$ $$4x^2+9x-9=0$$ $$(x+3)(4x-3)=0$$ $$x=-3 \text{ or } x=\frac{3}{4}$$ Or $$\frac{3}{2x+1}+\frac{2}{5}=\frac{2}{3x-1}$$ $$\frac{15+2(2x+1)}{5(2x+1)}=\frac{2}{3x-1}$$ $$\frac{4x+17}{10x+5}=\frac{2}{3x-1}$$ $$(4x+17)(3x-1)=2(10x+5)$$ $$12x^2+47x-17=20x+10$$ $$12x^2+27x-27=0$$ $$4x^2+9x-9=0$$ $$(x+3)(4x-3)=0$$ $$x=-3 \text{ or } x=\frac{3}{4}$$	**Scale 15D (0, 4, 7, 11,15)** *Low Partial Credit:* - CD or partial CD identified - Cross multiply on LHS - Multiplies one term correctly by one of the denominators - $x=-3$ **or** $x=\frac{3}{4}$ substituted and justified as a solution *Mid Partial Credit:* - Equation without fractions *High Partial Credit:* - Relevant quadratic in the form: $ax^2+bx+c=0$ <u>*Note*</u>: No quadratic $\Rightarrow$ low partial credit at most, except in the case where the candidate has reached the mid partial stage

Q2	Model Solution – 25 Marks	Marking Notes
(a) (i)	(0, 1) (2, 9) 	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - 1 point on line found *High Partial Credit:* - 2 points on line found - 1 point found and plotted (apart from (0, 1) and (2, 9)) *Full Credit −1*: - Freehand graph drawn
(a) (ii)	$g(1{\cdot}9) = 4(1{\cdot}9) + 1 = 8{\cdot}6$ $f(1{\cdot}9) = 3^{1{\cdot}9} = 8{\cdot}06$ $f(x) < g(x)$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - $g(1{\cdot}9)$ written or found - $f(1{\cdot}9)$ written or found *High Partial Credit:* - $g(1{\cdot}9)$ and $f(1{\cdot}9)$ found

(b)	To Prove: $3^n \geq 4n + 1$ for $n \geq 2$ $P(2)$: $3^2 \geq 4(2) + 1$ $9 \geq 9$, True Assume $P(n)$ is true for $n = k$, Now prove $P(n)$ is true for $n = k + 1$ $P(k)$: $3^k \geq 4k + 1$ for $k \geq 2$ $P(k+1)$: $3^{k+1} \geq 4(k+1) + 1$ $3^{k+1} \geq 4k + 5$ *Proof*: $P(k) \times 3$: $3^{k+1} \geq 3(4k+1)$ $= 12k + 3$ $\Rightarrow$ $3^{k+1} \geq 4k + 5$ since $4k + 5 < 12k + 3$ for $k \geq 2$ True for $n = k + 1$ provided true for $n = k$ but true for $n = 2$ $\therefore$ True for all $n \geq 2, n \in \mathbb{N}$.	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit*: - Step $P(2)$ - $P(k)$ or $P(k+1)$ with incorrect inequality sign *Mid Partial Credit*: - Any two of $P(2)$, $P(k)$ or $P(k+1)$ *High Partial Credit*: - Uses Step $P(k)$ to prove Step $P(k+1)$ *Full Credit −1*: - Omits conclusion but otherwise correct <u>*Note*</u>: Accept Step $P(2)$, Step $P(k)$, Step $P(k+1)$ in any order <u>*Note*</u>: Accept $f(k) \geq g(k)$, $k \geq 2$ for Step $P(k)$

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$(3x+4)(y-3)$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* - Any relevant factorisation
(b)	$3xlnx - 9x + 4lnx - 12 =$ $3x(lnx - 3) + 4(lnx - 3) =$ $(3x+4)(lnx-3)$ $3x + 4 = 0 \Rightarrow x = -\frac{4}{3}$ (not possible) $lnx - 3 = 0$ $lnx = 3$ $x = e^3$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant factorisation of $g(x)$ - Trial and improvement with at least two values tested - Substitutes $20 \leq x \leq 20{\cdot}1$ - $y = lnx$ *Mid Partial Credit* - Expression fully factorised *High Partial Credit:* - $lnx = 3$ *Full Credit –1:* - Both solutions presented <u>Note</u>: Accept $x = 20{\cdot}1$ for $x = e^3$ in the last line of the solution <u>Note</u>: If no reference is made to $3x + 4$ in the solution, then award high partial credit **at most**

(c)	$g'(x) = 3x\left(\frac{1}{x}\right) + (3)lnx - 9 + 4\left(\frac{1}{x}\right)$ $g'(e) = 3(e)\left(\frac{1}{e}\right) + (3)\,ln(e) - 9 + 4\left(\frac{1}{e}\right)$ $g'(e) = 3 + 3 - 9 + \frac{4}{e} = -1{\cdot}53$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant differentiation - $g(e)$ evaluated correctly to at least 2 decimal places *Mid Partial Credit* - Expression fully differentiated - Product rule not applied but finishes correctly *High Partial Credit:* - Derivative fully substituted

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$\frac{4x^4}{4} - \frac{6x^2}{2} + 10x + C$ $x^4 - 3x^2 + 10x + C$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Any relevant integration *High Partial Credit:* - 3 correct terms
(b) (i)	$\int (6x^2 - 54x + 109)\, dx$ $= 2x^3 - 27x^2 + 109x + C = f(x)$ $(2, 0) \in f(x)$ $2(2)^3 - 27(2)^2 + 109(2) + C = 0$ $2(8) - 27(4) + 218 + C = 0$ $16 - 108 + 218 + C = 0$ $16 + 110 + C = 0$ $126 + C = 0$ $C = -126$ $\therefore f(x) = 2x^3 - 27x^2 + 109x - 126$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant integration *Mid Partial Credit* - 3 correct terms *High Partial Credit:* - Relevant equation in C <u>*Note*</u>*:* Must integrate or indicate integration to gain any credit

(b) **(ii)**	2 is a root $\Rightarrow (x-2)$ is a factor $2x^3 - 27x^2 + 109x - 126 = 0$ $2x^2(x-2) - 23x(x-2) + 63(x-2)$ $2x^2 - 23x + 63 = 0$ $(2x - 9)(x-7) = 0$ $x = 4{\cdot}5$ or $x = 7$ $\therefore B(4{\cdot}5, 0)$ and $C(7, 0)$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - 2 identified as root - 0 given as the y co-ordinate - Sets up equation - Any integer fully substituted in $f(x)$ fully worked - $(x-2)$ is a factor - Sets up the correct equation *Mid Partial Credit* - Division completed with no remainder - 7 identified as a root - One coordinate pair found *High Partial Credit:* - x values found from factors

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$3 - 2i =$ other root $-p = (3 + 2i) + (3 - 2i) = 6$ $p = -6$ $q = (3 + 2i)(3 - 2i) = 13$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Second root identified *High Partial Credit:* - Sum and product of roots formulated into equations for p and q - p or q found correctly
	Or	
	$(3 + 2i)^2 + p(3 + 2i) + q = 0$ $5 + 12i + 3p + 2pi + q = 0$ $2p = -12 \Rightarrow p = -6$ $5 + 3p + q = 0 \ \Rightarrow q = 13$	*Low Partial Credit:* - Root substituted into equation - Any correct substitution *High Partial Credit:* - Real and imaginary terms formulated into equations for p and for q
	Or	
	$\dfrac{-p \pm \sqrt{p^2 - 4q}}{2} = 3 \pm 2i$ $-p \pm \sqrt{p^2 - 4q} = 6 \pm 4i$ $-p = 6$ $\therefore p = -6$ $\sqrt{4q - p^2} = 4$ $4q - p^2 = 16$ $4q - (-6)^2 = 16$ $4q = 52$ $\therefore q = 13$	*Low Partial Credit:* - Some substitution into quadratic formula *High Partial Credit:* - Finds p - Full substitution into quadratic formula and equated to either root.

(b) **(i)**	$\lvert v \rvert = \sqrt{4+12} = 4$ $\theta = 300°$ $v = 4(\cos 300° + i \sin 300°)$ Or $\lvert v \rvert = \sqrt{4+12} = 4$ $\theta = \frac{5\pi}{3}$ $v = 4\left(\cos\frac{5\pi}{3} + i \sin\frac{5\pi}{3}\right)$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Correct plot on the Argand diagram - Some use of Pythagoras to find modulus - Some use of trigonometry to find argument *High Partial Credit:* Modulus or argument found Note: Accept $4\left(\cos -\frac{\pi}{3} + i \sin -\frac{\pi}{3}\right)$ and $4(\cos -60° + i \sin -60°)$
(b) **(ii)**	$w = \pm v^{\frac{1}{2}}$ $w = \pm 2(\cos 300 + i \sin 300)^{\frac{1}{2}}$ $w = \pm 2(\cos 150 + i \sin 150)$ $w = \pm(-\sqrt{3} + i)$ $w = -\sqrt{3} + i$ or $\sqrt{3} - i$ Or $w = [4(\cos(300 + 360n) + i\sin(300 + 360n)]^{\left(\frac{1}{2}\right)}$ $w = 4^{\frac{1}{2}}[\cos(150 + 180n) + i\sin(150 + 180n)]$ $\underline{n = 0}$ $w = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\sqrt{3} + i$ $\underline{n = 1}$ $w = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt{3} - i$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - w written in polar form with index - Some use of De Moivre's Theorem - $w = v^{\frac{1}{2}}$ *High Partial Credit:* - De Moivre's theorem applied to w - One solution found - Solutions in polar form Note: Accept candidates answer from (b)(i)

<table>
<tr><th>Q6</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>$x + 5x = \sqrt{128} + \sqrt{32}$
$6x = 8\sqrt{2} + 4\sqrt{2}$
$6x = 12\sqrt{2}$
$x = 2\sqrt{2}$
Or
$x - \sqrt{32} = \sqrt{128} - 5x$
$\left(x - \sqrt{32}\right)^2 = \left(\sqrt{128} - 5x\right)^2$
$\left(x - 4\sqrt{2}\right)^2 = \left(8\sqrt{2} - 5x\right)^2$
$x^2 - 8\sqrt{2}x + 32 = 128 - 80\sqrt{2}x + 25x^2$
$x^2 - 3\sqrt{2}x + 4 = 0$
$\left(x - \sqrt{2}\right)\left(x - 2\sqrt{2}\right) = 0$
$x = \sqrt{2}$ or $x = 2\sqrt{2}$
Check solutions:
$x = \sqrt{2}$
$\sqrt{2} - \sqrt{32} = \sqrt{128} - 5\sqrt{2}$
$-3\sqrt{2} = 3\sqrt{2}$ (False)
Solution: $x = 2\sqrt{2}$</td><td>Scale 10C (0, 4, 7, 10)
Low Partial Credit:
- Any relevant transposing
- $\sqrt{32}$ or $\sqrt{128}$ in the form a$\sqrt{2}$

High Partial Credit
- x term isolated in equation

Low Partial Credit:
- $\sqrt{32}$ or $\sqrt{128}$ in the form $a\sqrt{2}$
- Any relevant multiplication

High Partial Credit:
- LHS and RHS squared correctly
- Solution not in the form $a\sqrt{2}$

Full Credit –1:
- Both solutions presented

<u>Note</u>: If $\sqrt{128}$ and $\sqrt{32}$ are converted to decimals, then award low partial credit at most</td></tr>
<tr><td>(a)
(ii)</td><td>$\sqrt{32k^2}, \sqrt{128k^2}, \sqrt{98k^2}, \sqrt{50k^2}$
$4\sqrt{2}\,k,\quad 8\sqrt{2}\,k,\quad 7\sqrt{2}\,k,\quad 5\sqrt{2}\,k$
$4\sqrt{2}\,k,\quad 5\sqrt{2}\,k,\quad 7\sqrt{2}\,k,\quad 8\sqrt{2}\,k$
$\text{Mean} = \frac{24\sqrt{2}\,k}{4} = 6\sqrt{2}\,k$
$\text{Median} = 6\sqrt{2}\,k$</td><td>Scale 5C (0, 2, 3, 5)
Low Partial Credit:
- List in ascending or descending order
- Any term written in the form $a\sqrt{2}k$ or in the form $a\sqrt{2k^2}$

High Partial Credit:
- Mean or median found
- Verified for a particular value of k

<u>Note</u>: If decimals are used then award low partial credit at most</td></tr>
</table>

(b)	Assume $\sqrt{2}$ is rational i.e. $\sqrt{2} = \frac{p}{q}$ where p and q have no common factors (simplest form) $\Rightarrow 2 = \frac{p^2}{q^2}$ $\Rightarrow 2q^2 = p^2$ $\Rightarrow p^2$ is even $\Rightarrow$ p is even $\Rightarrow p = 2k$ for some $k \in \mathbb{Z}$ $2q^2 = p^2$ becomes $2q^2 = 4k^2$ $\Rightarrow q^2 = 2k^2$ $\Rightarrow q^2$ is even $\Rightarrow$ q is even $\Rightarrow q = 2m$ for some $m \in \mathbb{Z}$ $\therefore \sqrt{2} = \frac{p}{q} = \frac{2k}{2m}$ $\Rightarrow$ common factor of 2 (contradiction) $\therefore \sqrt{2}$ cannot be rational.	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - $\sqrt{2} = \frac{p}{q}$ or similar *Mid Partial Credit* - deduces that p is even or equivalent - $p = 2k$ or equivalent deduced - $p^2 = 2q^2$ *High Partial Credit:* - $q = 2m$ or equivalent deduced

Section B		
Q7	**Model Solution – 45 Marks**	**Marking Notes**
(a) (i)	<table><tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr><tr><td>Fraction</td><td>$\frac{1}{3}$</td><td>$\frac{2}{9}$</td><td>$\frac{4}{27}$</td><td>$\frac{8}{81}$</td><td>$\frac{16}{243}$</td></tr></table>	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - 1 correct fraction given in table - 1 correct denominator - 1 correct numerator *High Partial Credit:* - 2 correct fractions given in table - All numerators correct - All denominators correct
(a) (ii)	$a = \frac{1}{3}\ \ r = \frac{2}{3}$ $S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{\frac{1}{3}\left(1-\left(\frac{2}{3}\right)^n\right)}{1-\frac{2}{3}}$ $S_n = 1 - \left(\frac{2}{3}\right)^n$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - S_n formula with some substitution - Correct a or correct r identified *High Partial Credit:* - S_n formula fully substituted
(a) (iii)	Infinite Geometric Series $a = \frac{1}{3}\ \ r = \frac{2}{3}$ $S_\infty = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{2}{3}} = 1$ Or $\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1-\left(\frac{2}{3}\right)^n\right) = 1$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - S_∞ indicated - Correct a or correct r identified *High Partial Credit:* - S_∞ fully substituted <u>Note</u>: If $\lvert r\rvert > 1$, then award low partial credit at most

<table>
<tr>
<td>(b)
(i)</td>
<td>
<table>
<tr><td>Label</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr>
<tr><td>End-point</td><td>$\frac{2}{3}$</td><td>$\frac{2}{9}$</td><td>$\frac{7}{9}$</td><td>$\frac{8}{9}$</td><td>$\frac{7}{27}$</td><td>$\frac{25}{27}$</td></tr>
</table>
</td>
<td>
Scale 10C (0, 4, 7, 10)
Low Partial Credit:
- 1 correct fraction given in table
- All denominators correct

High Partial Credit:
- 4 correct fractions given in table
</td>
</tr>
<tr>
<td>(b)
(ii)</td>
<td>
It is the end point (start point) of a segment
Or
$\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}=\frac{20}{81}$
$\frac{6}{27}$ ∘ ∘ $\boldsymbol{E}\,\frac{7}{27}$
$\frac{18}{81}$ $\frac{19}{81}$ $\frac{20}{81}$ $\frac{21}{81}$
Or
$\frac{7}{27}-\frac{1}{81}=\frac{20}{81}$ is a point in the Cantor Set
</td>
<td>
Scale 5B (0, 2, 5)
Mid Partial Credit:
- Relevant but incomplete reason given
- Sum of fractions $=\frac{20}{81}$
</td>
</tr>
<tr>
<td>(b)
(iii)</td>
<td>
$S_\infty=\frac{a}{1-r}=\frac{\frac{1}{3}}{1-\left(-\frac{1}{3}\right)}=\frac{1}{4}$
Or
$\frac{1}{3}+\frac{1}{27}+\frac{1}{243}+\cdots=\frac{\frac{1}{3}}{1-\frac{1}{9}}$
$=\frac{3}{8}$
$-\left(\frac{1}{9}+\frac{1}{81}+\frac{1}{729}+\cdots\right)=-\left(\frac{\frac{1}{9}}{1-\frac{1}{9}}\right)$
$=-\frac{1}{8}$
$S_\infty=\frac{3}{8}-\frac{1}{8}$
$=\frac{1}{4}$
</td>
<td>
Scale 10C (0, 4, 7, 10)
Low Partial Credit:
- S_∞ indicated
- S_∞formula with some substitution
- Correct a or correct r

High Partial Credit:
- S_∞ formula fully substituted
</td>
</tr>
</table>

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$r(20) = 22500\cos\left(\frac{\pi}{26}(20)\right) + 37500$ $= 22500\cos\left(\frac{20\pi}{26}\right) + 37500$ $= €20658{\cdot}51$ $\approx €20659$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Any relevant substitution - $r(20)$ or $t = 20$ *High Partial Credit:* - Correct substitution *Full Credit –1:* - Uses degrees as unit of measurement, giving an answer of €59980
(b)	$22500\cos\left(\frac{\pi}{26}t\right) + 37500 = 26250$ $22500\cos\left(\frac{\pi}{26}t\right) = -11250$ $\cos\left(\frac{\pi}{26}t\right) = -\frac{1}{2}$ $\frac{\pi}{26}t = \frac{2\pi}{3}$ and $\frac{\pi}{26}t = \frac{4\pi}{3}$ $t = \frac{52}{3}$ and $t = \frac{104}{3}$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Equation formed - Trial and improvement with at least two values tested *Mid Partial Credit:* - Equation simplified to: $\cos\left(\frac{\pi}{26}t\right) = -\frac{1}{2}$ - Equation simplified to: $\cos\left(\frac{\pi}{26}t\right) = -\frac{11250}{22500}$ *High Partial Credit:* - 1 correct solution to equation found

(c)	$r'(t) = 22500\left[-\sin\left(\frac{\pi}{26}t\right)\right]\left(\frac{\pi}{26}\right)$ $= -\frac{11250}{13}\pi\left[\sin(\frac{\pi}{26}t)\right]$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Some relevant differentiation *High Partial Credit:* - Chain rule applied
(d)	$r'(30) = -\frac{11250}{13}\pi\left[\sin(\frac{\pi}{26}(30))\right]$ $= 402{\cdot}164\pi$ $= 1263{\cdot}44$ > 0 $\Rightarrow$ Increasing	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* - Some relevant substitution into answer from (c) - $r'(t) > 0$ - $\frac{dy}{dx} > 0$ *High Partial Credit:* - $r'(30)$ found but no conclusion or incorrect conclusion *Note*: If calculus is not used then award no credit for the solution

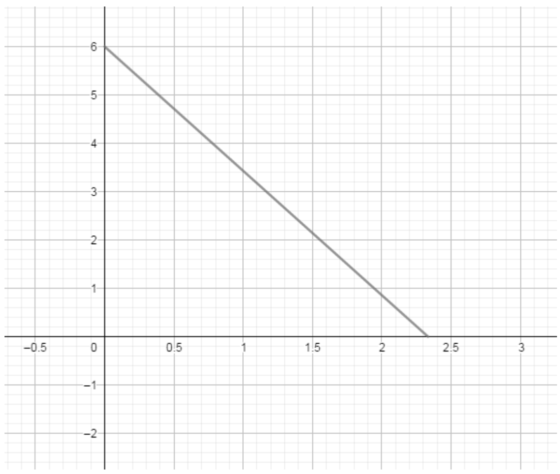
(e)	$-\frac{11250}{13}\pi\left[\sin\left(\frac{\pi}{26}t\right)\right]=0$ $\sin\left(\frac{\pi}{26}t\right)=0$ $\frac{\pi}{26}t=0$ and $\frac{\pi}{26}t=\pi$ $t=0$ and $t=26$ $r''(t)=-\frac{11250}{13}\pi\left[\cos\left(\frac{\pi}{26}t\right)\right]\left(\frac{\pi}{26}\right)$ $t=0$: $r''(t)<0 \Rightarrow$ Max $t=26$: $r''(t)>0 \Rightarrow$ Min Or Range: $[37500-22500, 37500+22500]$ $=[15{,}000, 60{,}000]$ $22500\cos\left(\frac{\pi}{26}t\right)+37500=15000$ $22500\cos\left(\frac{\pi}{26}t\right)=15000-37500$ $22500\cos\left(\frac{\pi}{26}t\right)=-22500$ $\cos\left(\frac{\pi}{26}t\right)=-1$ $\frac{\pi}{26}t=\pi$ $\therefore t=26$ $r''(26)=-\frac{11250}{13}\pi\left[\cos\left(\frac{\pi}{26})26)\right)\right]\left(\frac{\pi}{26}\right)$ >0 $\Rightarrow$ Min	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* - $r'(t)=0$ - $\frac{dy}{dx}=0$ - States $r''(t)>0$ at minimum value - $t=26$ and no further work *Mid Partial Credit* - $t=0$ or $t=26$ found with supporting work - $r''(t)$ found *High Partial Credit:* - $t=26$ found with supporting work and $r''(t)$ found (including use of chain rule)

Q9	Model Solution – 55 Marks	Marking Notes
(a) (i)	$= 2(x) + 2(y) + \frac{1}{2}(2\pi)(x)$ $= 2x + 2y + \pi x$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Some relevant substitution into perimeter formula - Circumference of circle of radius x found i.e. $2\pi x$ *High Partial Credit:* - Two of the three terms found
(a) (ii)	$2x + 2y + \pi x = 12$ $2y = 12 - 2x - \pi x$ $y = \frac{12 - 2x - \pi x}{2}$ $y = \frac{12 - (2 + \pi)x}{2}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Some relevant substitution into equation *High Partial Credit:* - y term isolated correctly in equation Note: Accept candidates answer from (a)(i) provided it doesn't oversimplify the work. Note: Must draw a relevant conclusion from incorrect work

(b) (i) + (ii)

Table and Graph

x	0	$\dfrac{12}{2+\pi}$
$y=\dfrac{12-(2+\pi)x}{2}$	6	0



Scale 15D (0, 4, 7, 11, 15)

Low Partial Credit:
- One correct table entry
- One correct plot of incorrect point

Mid Partial Credit:
- 2 table entries correct
- 2 incorrect points plotted and joined

High Partial Credit:
- 2 correct points plotted but not joined with correct table entries

Full Credit −1:
- Two correct points plotted and joined but the function is not graphed in the stated domain

<u>*Note*</u>: Accept $2{\cdot}25 \le x \le 2{\cdot}5$ for x−intercept

(b) **(iii)**	$y = \dfrac{12 - (2+\pi)x}{2}$ $y = 6 - \left(\dfrac{2+\pi}{2}\right)x$ $m = -\left(\dfrac{2+\pi}{2}\right)$ $m = -2{\cdot}57$ Or $m = \dfrac{0-6}{\frac{12}{2+\pi} - 0}$ $m = -\left(\dfrac{2+\pi}{2}\right)$ $m = -2{\cdot}57$ Intepretation: For each 1m rise in the radius of the semi-circle, the height of the rectangle falls by approximately 2·57 m	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - Some substitution into slope formula - Slope isolated in the equation of the line formula - $\frac{dy}{dx}$ - $\frac{rise}{run}$ with some relevant substitution - Some effort at differentiation *High Partial Credit:* - Slope found *Note*: Accept $-2{\cdot}7 \leq$ slope $\leq -2{\cdot}5$ from relevant work
(c) **(i)**	$a = 2xy + \dfrac{\pi x^2}{2}$ $= \dfrac{2x[(12 - (2+\pi)x]}{2} + \dfrac{\pi x^2}{2}$ $= \dfrac{24x - 4x^2 - 2\pi x^2}{2} + \dfrac{\pi x^2}{2}$ $= \dfrac{24x - (\pi + 4)x^2}{2}$	0. **Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* - area of rectangle correct - area of semi-circle correct *High Partial Credit:* - Both areas correct in terms of x and added

(c) (ii)	$a(x) = \frac{1}{2}(24x - (\pi + 4)x^2)$ $a'(x) = \frac{1}{2}(24 - 2(\pi + 4)x)$ $= 12 - (\pi + 4)x$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* - Some correct differentiation
(c) (iii)	$a'(x) = 0$ $12 - (\pi + 4)x = 0$ $(\pi + 4)x = 12$ $x = \frac{12}{\pi+4}$ (1·68) $y = \frac{12-(2+\pi)x}{2}$ $(= \frac{12-(5{\cdot}14)\ 1{\cdot}68}{2} \approx 1{\cdot}68)$ $= \frac{12-(2+\pi)(\frac{12}{\pi+4})}{2}$ $= \frac{12(\pi+4)-(2+\pi)(12)}{2(\pi+4)}$ $= \frac{12\pi+48-24-12\pi}{2(\pi+4)}$ $= \frac{24}{2(\pi+4)}$ $= \frac{12}{\pi+4}$ $= x$ Area Max when height equals the radius	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* - $a'(x)$ used - States $\frac{dy}{dx} = 0$ *Mid Partial Credit* - Value of x at maximum found *High Partial Credit:* - Value of y at maximum fully substituted