Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2018

Mathematics

Ordinary Level

Paper 1

Solutions and Marking scheme

300 marks

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 3, 5	0, 3, 4, 5	0,2,3,4,5	
10 mark scales	0, 10	0, 5, 10	0, 3, 8, 10	0, 2, 6, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 4, 12, 15	0, 3, 7, 13, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)	10D
(b)	5C
(c)	10C

Question 2

(a)	15D
(b)	5D
(c)	5C

Question 3

(a)	10D
(b)	15D

Question 4

(a)	5B
(b)	5C
(c)	5C
(d)	10C

Question 5

(a)	5C
(b)	10C
(c)	10C

Question 6

(a)	10C
(b)	15D

Section B

Question 7

(a)	5B
(b)	10C
(c)	10C
(d)	10C
(e)	5C
(f)(i)	5C
(f)(ii)	10C

Question 8

(a)	10C
(b)	10C
(c)	5C
(d)(i)	5C
(d)(ii)	10C
(e)(i)	10C
(e)(ii)	5C
(e)(iii)	5C
(e)(iv)	5D

Question 9

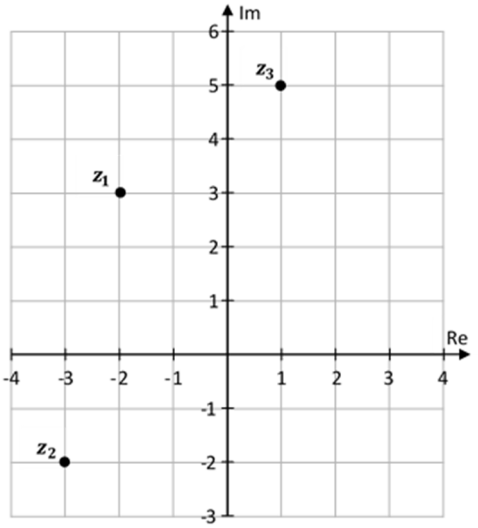
(a)(i)	5B
(a)(ii)	5C
(b)(i)	5B
(b)(ii)	15C

Note: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus for example, in scale 10C, 9 marks may be awarded.
Rounding and units penalty to be applied only once in each part (a), (b), (c) etc.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

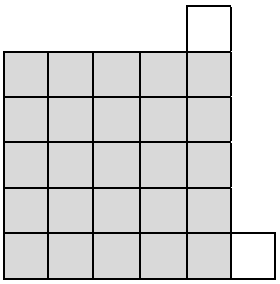
Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	Contract A $35000 + 400000 \times 0{\cdot}02$ $= €43\,000$ Contract B $30000 + 400000 \times 0{\cdot}03$ $= €42\,000$	**Scale 10D (0, 2, 6, 8, 10)** *Low Partial Credit:* • Use of 2% or 3% *Mid Partial Credit* • 8000 or 12000 or both *High Partial Credit:* • One contract correct **Note**: Accept correct answer without work
(b)	$50000 - 35000 = 15000$ $(15000 \div 0{\cdot}02) = €750\,000$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $50000 - 35000$ formulated or 15000 *High Partial Credit:* • $15000 \times 0{\cdot}02$ or equivalent $= 300$ **Note**: Accept correct answer without work
(c)	$35000 + 0{\cdot}02x = 30000 + 0{\cdot}03x$ $x = €500\,000$ **Or** 1% = $€5000$ Answer = $€500\,000$	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • $35000 - 30000$ or 5000 written • correct answer without work *High Partial Credit:* • $35000 + 0{\cdot}02x = 30000 + 0{\cdot}03x$ • $\frac{x}{100} = 5000$

Q2	Model Solution – 25 Marks	Marking Notes
(a)	 $z_3 = -2 + 3i + 3 + 2i = 1 + 5i$	**Scale 15D (0, 3, 7, 13, 15)** *Low Partial Credit:* • $z_3 = z_1 - z_2$ with some substitution • 1 or 2 correct plots (z_1 and/or z_2) without labels *Mid Partial Credit* • z_3 correct, with or without plotting • 2 correct plots (z_1 and z_2) with labels *High Partial Credit:* • Plot of incorrect z_3 and 2 correct plots labelled • Mixes up real and imaginary axes but plots and labels 3 points 'correctly'
(b)	$\sqrt{13} + \sqrt{13} \neq \sqrt{26}$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • $\lvert z_1 \rvert$ or $\lvert z_2 \rvert$ or $\lvert z_3 \rvert$ formulated with some substitution • Correct formula *Mid Partial Credit:* • $\lvert z_1 \rvert$ or $\lvert z_2 \rvert$or $\lvert z_3 \rvert$ correct *High Partial Credit:* • $\lvert z_1 \rvert$ and $\lvert z_2 \rvert$ and $\lvert z_3 \rvert$ correct but no conclusion or incorrect conclusion
(c)	$\frac{(-2+3i)(-3+2i)}{(-3-2i)(-3+2i)}$ $= 0 - 1i$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Some correct substitution • Conjugate identified • Some multiplication above and below by same number, even if incorrect conjugate • correct answer without work *High Partial Credit:* • $\frac{(-2+3i)(-3+2i)}{(-3-2i)(-3+2i)}$

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$\frac{7 \pm \sqrt{(-7)^2 - 4(2)(-3)}}{4}$ $= 3{\cdot}89 \quad \text{or} \quad -0{\cdot}39$	**Scale 10D (0, 2, 6, 8, 10)** *Low Partial Credit:* • correct roots formula • a or b or c identified • attempt at factorising • correct answer without work *Mid Partial Credit:* • formula fully substituted *High Partial Credit:* • one root calculated • $x = \frac{7 \pm \sqrt{73}}{4}$ and stops
(b)	$-2a - 3b = -15$ $\underline{15a + 3b = -24}$ 13a = -39 $a = -3$ and $b = 7$	**Scale 15D (0, 3, 7, 13, 15)** *Low Partial Credit:* • effort to equate a or b coefficient in both equations • correct answer without work *Mid Partial Credit:* • eliminates one unknown *High Partial Credit:* • 1 unknown found

<table>
<tr><th>Q4</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>  </td><td>Scale 5B (0, 3 ,5)
Partial Credit:
• correct diagram, no shading
• 5^2 written</td></tr>
<tr><td>(b)</td><td colspan="2"><table>
<tr><th>Pattern number (n)</th><th>Number of Grey Tiles</th><th>Number of White Tiles</th><th>Total Number of Tiles</th></tr>
<tr><td>1</td><td>4</td><td>2</td><td>6</td></tr>
<tr><td>2</td><td>9</td><td>2</td><td>11</td></tr>
<tr><td>3</td><td>16</td><td>2</td><td>18</td></tr>
<tr><td>4</td><td>25</td><td>2</td><td>27</td></tr>
<tr><td>5</td><td>36</td><td>2</td><td>38</td></tr>
</table></td></tr>
<tr><td></td><td></td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
• At least 1 correct new entry

High Partial Credit:
• At least 6 correct or consistent new entries</td></tr>
</table>

(c)	$(n+1)^2+2$ n^2+2n+3 **Or** $n=1 \quad \Rightarrow \quad b+c=5$ $n=2 \quad \Rightarrow 2b+c=7$ $b=2$ $c=3$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $(n)^2+2$ written and stops or continues • Effort at substitution to derive one equation in a and b • correct answer without work *High Partial Credit*: • $(n+1)^2+2$ • 2 relevant equations derived
(d)	$n^2+2n+3=443$ $n^2+2n-440=0$ $n=20$ **Or** $6, 11, 18, \ ., \ ., \ ., \quad 443$ $n=20$	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • $n^2+2n+3=443$ • effort at trial and error involving any term beyond T_5 *High Partial Credit:* • $n^2+2n-440=0$ • $(21)^2+2=443$ **Note**: Accept $n=20$ without work for full marks

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$A = (\ 0,\ 6\ )$ $B = (-2,\ 0\ )$ $C = (\ 3,\ 0\ )$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • 1 point correctly identified *High Partial Credit*: • 2 points correctly identified • mixes up axes consistently for the 3 points
(b)	$f(x) = ax^2 + bx + c$ $c = 6$ $0 = (-2)^2a + b(-2) + 6$ $-6 = 4a - 2b$ $0 = (3)^2a + b(3) + 6$ $-6 = 9a + 3b$ --- $-6 = 9a + 3b$ $-6 = 4a - 2b$ $a = -1,\ \ b = 1$ **Or** $-(x+2)(x-3)$ $= -x^2 + x + 6$	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • $c = 6$ • substitution of *A* or *B* or *C* into given equation or $ax^2 + bx + c$ • 1 factor written *High Partial Credit*: • substitution of any 2 of *A*, *B*, or *C* into given equation worked out • substitution of *A*, *B*, and *C* into $ax^2 + bx + c$ • $\pm(x+2)(x-3)$ **Note**: *A*, *B* and *C* substituted into $-x^2 + x + 6$ and fully verified merits full marks
(c)	$f(x) = -x^2 + x + 6$ $f'(x) = -2x + 1 = 0$ $x = \frac{1}{2}$ $-\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6 = 6{\cdot}25$	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • any effort at differentiation • states $f'(x) = 0$ *High Partial Credit*: • $x = \frac{1}{2}$

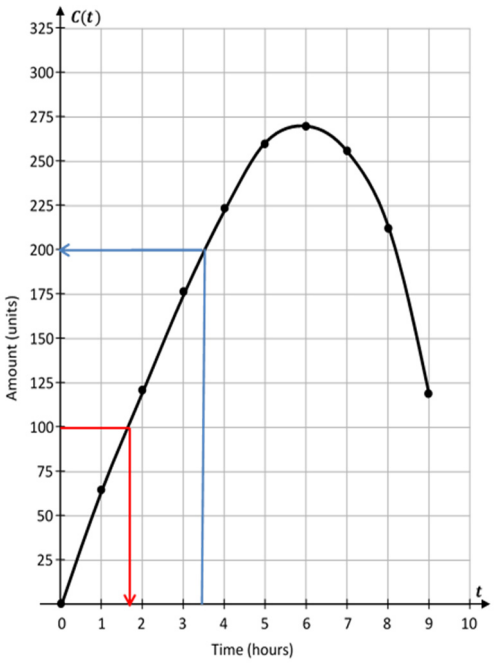
Q6	Model Solution – 25 Marks	Marking Notes
(a)	$(x+5)(3x-4)-3(x^2+2)+4=0$ $3x^2-4x+15x-20-3x^2-6+4=0$ $11x=22$ $x=2$	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • any correct multiplication • correct answer without work *High Partial Credit*: • multiplication fully correct • error in expanding brackets but finishes correctly
(b)	$(2x)(x+3)\left(\frac{5}{x+3}-\frac{1}{x}\right)=\frac{1}{2}(2x)(x+3)$ $10x-2(x+3)=x(x+3)$ $x^2-5x+6=0$ $x=2 \quad \text{and} \quad x=3$	**Scale 15D (0, 3, 7, 13, 15)** *Low Partial Credit:* • common denominator identified • $5x-(x+3)$ or similar • correct answer without work, 2 and/or 3 • writes $\frac{-b\pm\sqrt{(b)^2-4ac}}{2a}$ *Mid Partial Credit*: • $10x-2(x+3)=x(x+3)$ or similar *High Partial Credit*: • $x^2-5x+6=0$ or equivalent

Section B		
Q7	**Model Solution – 55 Marks**	**Marking Notes**
(a)	37	**Scale 5B (0, 3, 5)** *Partial Credit:* • correct number of seats for any row between row 4 and row 9 (inclusive) **Note:** Accept 37 without work
(b)	$28 + (n-1)1 = 50$ $27 + n = 50$ $n = 23$ **Or** 50 – 37 = 13 13 + 10 = 23	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • any evidence of counting (e.g. listing 28, 29,…) • 13 or 22 without work • writes T_n = a + (n-1)d • a or d identified *High Partial Credit:* • 50 – 28 or 50 – 37 • $28 + (n-1)1 = 50$ **Note:** Accept 23 without work
(c)	$S_{23} = \frac{23}{2}[2(28) + (22)(1)]$ $S_{23} = 897$ (seats) **Or** 28 + 29 + . . . + 50 = 897	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • S_{n} formula written • any evidence of counting (e.g. listing 28 + 29 + 30 +…) • a or d identified *High Partial Credit:* • full listing with addition indicated but incorrect solution • S_{n} formula fully substituted **Note:** Accept 897 without work

(d)	$\frac{n}{2}(56 + n - 1) = 600$ $n^2 + 55n - 1200 = 0$ $n = 16{\cdot}7$ $S_{16} = 568$ $600 - 568 = 32$ people in next row **Or** [28 + 29 + . . .+ 43] = 568 600 – 568 = 32 16 rows and 32 in next **Or** ([28 + 29 + . . .+ 44] = 612 612 – 44 = 568 600 – 568 = 32 (17 –1 = 16) rows and 32 in next	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • S_n formula written • a or d identified • any evidence of counting (e.g. listing 28 + 29 + 30 +…) *High Partial Credit*: • $n^2 + 55n - 1200 = 0$ • $\frac{n}{2}(56 + n - 1) = 600$ • full listing to 16 or 17 terms with addition indicated but incorrect solution **Note:** Accept (16, 32) without work
(e)	$276(25) + 212(12)$ 6900 + 2544 = €9 444	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $276(25)$ • $212(12)$ *High Partial Credit*: • $276(25) + 212(12)$ formulated **Note**: Accept correct answer without work
(f) **(i)**	$\frac{752}{4}$ = 188 Children's tickets $188 \times 3 = 564$ Adult tickets	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • 4 • $\frac{752}{x}$, $x \neq 4, 1$ *High Partial Credit:* • 188 found or 564 **Note**: Accept correct answer without work
(f) **(ii)**	$188x + 564(2{\cdot}5x) = 17\,578$ $1598x = 17\,578$ $x = €11$	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • x, $2{\cdot}5x$ • correct answer without work *High Partial Credit*: • $188x + 564(2{\cdot}5x) = 17\,578$, or consistent with f(i)

<table>
<tr><th>Q8</th><th>Model Solution – 65 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$$C(4) = -(4^3) + 4{\cdot}5(4)^2 + 54(4)$$
$$= 224$$</td><td>Scale 10C (0, 3, 8, 10)
Low Partial Credit:
• Function with some substitution of 4

High Partial Credit:
• Function fully substituted</td></tr>
<tr><td>(b)</td><td colspan="2"><table>
<tr><td>$\boldsymbol{t}$ (Hours)</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>$\boldsymbol{C(t)}$ (Units)</td><td>0</td><td>57·5</td><td>118</td><td>175·5</td><td>224</td><td>257·5</td><td>270</td><td>255·5</td><td>208</td><td>121·5</td></tr>
</table></td></tr>
<tr><td></td><td></td><td>Scale 10C (0, 3, 8, 10)
Low Partial Credit:
• 1, 2 or 3 correct new entries

High Partial Credit:
• 4, 5 or 6 correct new entries</td></tr>
<tr><td>(c)</td><td> 
</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
• 1 to 4 correct plots

High Partial Credit:
• 5 to 9 correct plots
• all points correctly plotted but incorrect or no joining</td></tr>
</table>

(d) **(i)**	200 units	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • 3·5 shown on horizontal axis *High Partial Credit:* • correct construction shown but no indication of 200
(d) **(ii)**	1 hour 45 mins	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • 100 clearly indicated on vertical axis *High Partial Credit:* • correct construction shown but no indication of 1 hour 45 minutes
(e) **(i)**	$C'(t) = -3t^2 + 9t + 54$	**Scale 10C (0, 3, 8, 10)** *Low Partial Credit:* • any correct differentiation or indication of differentiation *High Partial Credit:* • 2 terms correctly differentiated
(e) **(ii)**	$C'(4) = -3(4)^2 + 9(4) + 54$ $= 42$ units / hour	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • some substitution of 4 into C' or writes $C'(4)$ *High Partial Credit:* • $C'(4) = -3(4)^2 + 9(4) + 54$ **Note**: Accept correct or consistent answer without work

(e) (iii)	$-3t^2 + 9t + 54 = 0$ $t^2 - 3t - 18 = 0$ $(t-6)(t+3) = 0$ $t = 6, \quad t = -3$ $t = 6$ Amount of drug $= 270$ units	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $C'(t) = 0$ • correct answer without work *High Partial Credit:* • $t = 6$ from calculus
(e) (iv)	$C'(7) = -3(7)^2 + 9(7) + 54$ $= -30$ units / hour Rate is negative so the amount of the drug is decreasing	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* • some substitution of 7 into C' or writes $C'(7)$ *Mid Partial Credit:* • $C'(7) = -3(7)^2 + 9(7) + 54$ *High Partial Credit:* • -30 units / hour

Q9	Model Solution – 30 Marks	Marking Notes
(a) (i)	$s = \sqrt{9{\cdot}8(2000)} = 140$	**Scale 5B (0, 3 ,5)** *Partial Credit:* • Any relevant substitution **Note**: Accept correct answer without work
(a) (ii)	$\frac{400000}{140} = 2857{\cdot}14$ sec $= 47{\cdot}62$ mins $= 48$ mins	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • use of answer to Part (a)(i) • correct answer without work • 400000 written *High Partial Credit*: • $\frac{400000}{140}$
(b) (i)	$s^2 = g \times d$ $d = \frac{s^2}{g}$	Scale 5B (0, 3, 5) *Low Partial Credit:* • effort to square both sides
(b) (ii)	$d = \frac{55^2}{9{\cdot}8}$ $d = 308{\cdot}67$ $d = 309$ **0r** $55 = \sqrt{9{\cdot}8 \times d}$ $d = \frac{55^2}{9{\cdot}8}$ $d = 308{\cdot}67$ $d = 309$	**Scale 15C (0, 4, 12, 15)** *Low Partial Credit:* • some substitution into either formula • correct answer without work *High Partial Credit:* • $d = \frac{55^2}{9{\cdot}8}$