Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2017

Model Solutions and Marking Scheme

Mathematics

Ordinary Level

Paper 1

300 marks

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5		
10 mark scales	0, 10	0, 5, 10	0, 4, 6, 10	0, 4, 6, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 6, 10, 15	0, 5, 7, 10, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.
Thus, for example, in scale 10C, 9 marks may be awarded.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)	10C
(b)	10C
(c)	5C

Question 2

(a)(i)	5B
(a)(ii)	5A
(b)	10D
(c)	5C

Question 3

(a)	10D
(b)	15D

Question 4

(a)	10C
(b)	15D

Question 5

(a)	10C
(b)	15D

Question 6

(a)	10C
(b)	15D

Section B

Question 7

(a)	5A
(b)	5C
(c)(i)	10C
(c)(ii)	5B
(c)(iii)	5B
(d)(i)	15C
(d)(ii)	5C

Question 8

(a)	5B
(b)	10C
(c)(i)	10C
(c)(ii)	5C
(c)(iii)	5B
(d)	10C

Question 9

(a)	10C
(b)	5B
(c)	10D
(d)	10C
(e)(i)	5C
(e)(ii)	10B
(e)(iii)	5C

Model Solutions & Marking Detailed Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$A = 30\,000(1 - 0{\cdot}15)^5$ $A = €13\,311{\cdot}16$ **Or** List Method 30000 25500 21675 18423·75 15660·19 13311·16	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • $30\,000 \times 15\%$ or equivalent • $0{\cdot}85$ • at least one correct substitution • correct formula written *High Partial Credit:* • $(0{\cdot}85)^5$ • 3 or 4 years correct by list method • fully correct substitution into formula but fails to finish • correct answer without work (13311)
(b)	$P(1 + 0{\cdot}03)^2 = 30\,000$ $P = €28\,277{\cdot}88$ **Or** List Method 30000 29126·21 28277·88 or 28277·87	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • $1{\cdot}03$ • at least 1 correct substitution • correct formula written • 2 unsuccessful efforts at trial and improvement *High Partial Credit:* • $\frac{30000}{1.03^2}$ • correct answer without work, 28277 or 28278
(c)	$26\,530{\cdot}2 = 25\,000(1 + i)^3$ $1 + i = 1{\cdot}02$ $i = 2\%$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $\frac{26530{\cdot}20}{25000}$ • $1530{\cdot}20$ • $\sqrt[3]{\ }$ • at least 1 correct substitution or correct formula written • correct answer without work *High Partial Credit:* • $1{\cdot}02$

Q2	Model Solution – 25 Marks	Marking Notes
(a) (i)	$(a = 3,\ b = -1)$	**Scale 5B (0, 2, 5)** *Partial Credit:* • $a = -1, \quad b = 3$ • one correct value only
(a) (ii)	Correct plot of $-1 + 2i$ on diagram	**Scale 5A (0, 5)**
(a) (iii)	$z_3 = \dfrac{(3 - i)(-1 - 2i)}{(-1 + 2i)(-1 - 2i)}$ $z_3 = -1 - i$	**Scale 10D (0, 4, 6, 8, 10)** *Low Partial Credit:* • at least 1 correct substitution *Mid Partial Credit:* • conjugate identified • some multiplication above and below by the same number, even if incorrect conjugate *High Partial Credit:* • correct multiplication using correct conjugate • answer not in correct form
(b)	$2z - 6(4 - 6i) = (-1 + i)(4 - 2i)$ $2z = 22 - 30i$ $z = 11 - 15i$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • any correct multiplication *High Partial Credit:* • multiplication fully correct

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$x = \frac{6 \pm \sqrt{36 - 4(3)(-8)}}{6}$ $x = 2{\cdot}91485 \text{ or } -0{\cdot}9485$ $x = 2{\cdot}9 \text{ or } -0{\cdot}9$	**Scale 10D (0, 4, 6, 8, 10)** *Low Partial Credit:* • correct roots formula • a, b, and c explicitly identified • attempt at factorising *Mid Partial Credit:* • formula fully substituted • $x = \frac{6 \pm \sqrt{-60}}{6}$ and stops or continues *High Partial Credit:* • one correct root • $x = \frac{6 \pm \sqrt{132}}{6}$ and stops
(b)	$\frac{dy}{dx} = 6x - 6 = 0$ $x = 1$ $y = -11$	**Scale 15D (0, 5, 7, 10, 15)** *Low Partial Credit:* • $\frac{dy}{dx}$ or $f'(x)$ *or* $f'(x) = 0$ • any correct differentiation • some work towards a graphical solution • correct answer without work *Mid Partial Credit:* • $6x - 6$ or $6x - 6 = 0$ *High Partial Credit:* • finds x value correctly **Note**: Accept correct turning point found graphically for full marks

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$11x - 5(2x - 1) = 3(6 - x) + 3$ $x = 4$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • any correct multiplication • correct answer without work *High Partial Credit:* • multiplication fully correct • error in expanding brackets but finishes correctly
(b)	$y = 2x - 5$ $x^2 + (2x - 5)^2 = 25$ $5x(x - 4) = 0$ $x = 0$ or $x = 4$ $y = -5$ or $y = 3$	**Scale 15D (0, 5, 7, 10, 15)** *Low Partial Credit:* • isolates one variable *Mid Partial Credit:* • substitution into quadratic *High Partial Credit:* • solve (2 values) for 1 variable • finds one correct couple from a quadratic **Note**: $(2x - 5)^2 = 4x^2 + 25$ and continues correctly merits MPC

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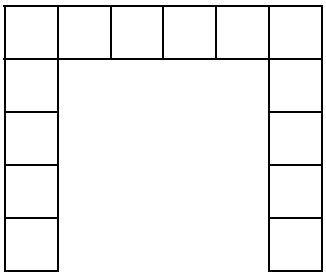
Q5	Model Solution – 25 Marks	Marking Notes
(a)	$A = \frac{3}{2}\begin{pmatrix}4 + 8 + 2[5\cdot8 + 7 + 6\cdot5 + 6 + \\ 4\cdot8 + 6 + 6\cdot5]\end{pmatrix}$ $A = 145\cdot8\ \text{m}^2$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • correct relevant area formula with some substitution *High Partial Credit:* • formula fully substituted • uses Simpsons Rule and finishes correctly
(b)	$145\cdot6 = \frac{4}{2}(4 + 8 + 2[6\cdot4 + 6\cdot9 + 6 + x + 6\cdot2])$ $4x = 19\cdot6$ $x = 4\cdot9$	**Scale 15D (0, 5, 7, 10, 15)** *Low Partial Credit:* • correct relevant area formula with some substitution *Mid Partial Credit:* • Formula fully substituted *High Partial Credit:* • x isolated from formula • uses Simpsons Rule and finishes correctly

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$150 + 0{\cdot}2(1000) + 0{\cdot}3(2000)$ €950 per week	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • $1000 \times 20\%$ or equivalent • 2000 *High Partial Credit:* • both commissions found, 200 and 600
(b)	$1160 - (150 + 200) = €810$ at 30% $\frac{810}{0{\cdot}3} = €2700$ higher sales $2700 + 1000 = €3700$ total sales **Or** 1160 – 950 = 210 210 = 30% 700 = 100% Total Sales = 3000 + 700 = €3700	**Scale 15D (0, 5, 7, 10, 15)** *Low Partial Credit:* • $1160 - 150$ • $1160 - 200$ *Mid Partial Credit:* • 810 • $\frac{x}{0{\cdot}3}$ *High Partial Credit:* • 2700

<table>
<tr><th>Q7</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>Pattern  4</td><td>Scale 5A (0, 5)</td></tr>
<tr><td>(b)</td><td><table><tr><th>Pattern number (n)</th><th>Number of Tiles</th></tr><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>8</td></tr><tr><td>3</td><td>11</td></tr><tr><td>4</td><td>14</td></tr><tr><td>5</td><td>17</td></tr></table></td><td>Scale 5C (0, 2, 3, 5)
Low Partial Credit:
• 1 or 2 correct new entries

High Partial Credit:
• 3 correct or consistent new entries</td></tr>
<tr><td>(c)
(i)</td><td>$T_n = a + (n-1)d$
$T_n = 5 + (n-1)3$
$T_n = 3n + 2$

or

$p + q = 5$
$2p + q = 8$
$p = 3 \qquad q = 2$</td><td>Scale 10C (0, 4, 6, 10)
Low Partial Credit:
• T_n formula written with some substitution
• 1 or 2 relevant equations in p & q
• $d = 3$ or $a = 5$ written

High Partial Credit:
• T_n formula fully substituted
• value of p or value of q found

Note: Accept $3n + 2$ without work for full marks</td></tr>
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(c) (ii)	$T_{20} = 62$	**Scale 5B (0, 2, 5)** *Partial Credit:* • formula with some substitution • listing of terms (>10) but T_{20} not identified or incorrectly identified **Note:** Accept correct answer without work
(c) (iii)	$3n + 2 = 290$ $n = 96$	**Scale 5B (0, 2, 5)** *Partial Credit:* • correct answer without work • equation (with formula) written • $3n + 2$ on its own • finds $T_{290} = 872$, but 872 without work is zero marks
(d) (i)	$S_n = \frac{n}{2}(10 + (n-1)3)$ $S_n = \frac{n}{2}(3n + 7)$ $S_n = \frac{3n^2 + 7n}{2}$	**Scale 15C (0, 6, 10, 15)** *Low Partial Credit:* • correct formula with some correct substitution • $a = 5$ and/or $d = 3$ *High Partial Credit:* • formula fully substituted
(d) (ii)	$S_{30} = \frac{3(30)^2+7(30)}{2}$ =1455 tiles	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • formula $= 1455$ • finds $S_n = 30$ • $a = 5$ and/or $d = 3$ • evidence of addition of terms in trial and improvement method *High Partial Credit:* • quadratic equation formed • substitutes correctly into either S_n formula

<table>
<tr><th>Q8</th><th>Model Solution – 45 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>€2000</td><td>Scale 5B (0, 2, 5)
Partial Credit:
• $x = 0$</td></tr>
<tr><td>(b)</td><td>$275x - x^2 - 2000 = 8350$
$-x^2 + 275x - 10350 = 0$
$x = 45,\ \ x = 230$

$x = 230$ is not possible as production limit is 200

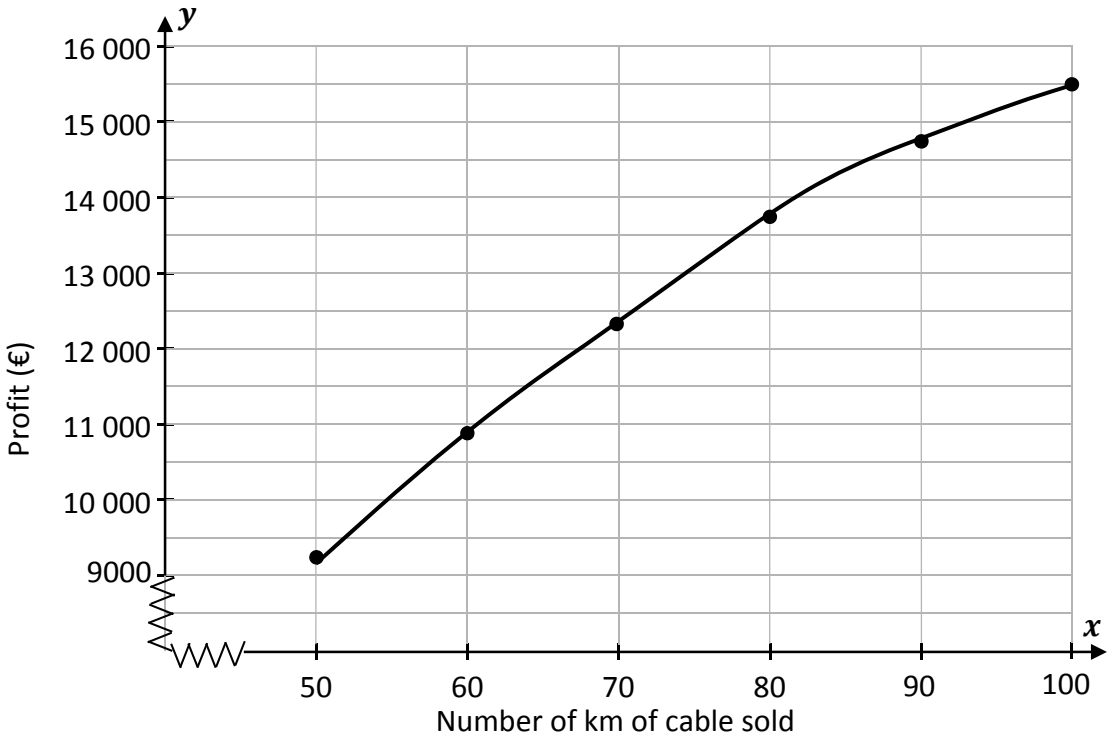
$x = 45$ km</td><td>Scale 10C (0, 4, 6, 10)
Low Partial Credit:
• quadratic equation written
• 45 without work

High Partial Credit:
• roots of quadratic

Note: 45 subbed in to get 8350 is full Marks</td></tr>
<tr><td>(c)
(i)</td><td><table><tr><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td><td>100</td></tr><tr><td>9250</td><td>10 900</td><td>12 350</td><td>13 600</td><td>14 650</td><td>15 500</td></tr></table></td><td>Scale 10C (0, 4, 6, 10)
Low Partial Credit:
• 1 or 2 correct new entries

High Partial Credit:
• 3 or 4 correct new entries</td></tr>
</table>

(c)
(ii)



(c) **(ii)**	Graph above	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • 1, 2 or 3 correct plots • bar chart *High Partial Credit:* • 4 or 5 correct plots • 6 correct plots but not joined
(c) **(iii)**	Lower 55 Upper 83 (Lines to be shown on diagram)	**Scale 5B (0, 2, 5)** *Partial Credit:* • evidence of use of graph • one correct range found • correct answers without work
(d)	$\frac{dP}{dx} = 275 - 2x = 105$ $2x = 170$ $x = 85$	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • effort at calculus *High Partial Credit:* • correct differentiation

Q9	Model Solution – 55 Marks	Marking Notes
(a)	$male = 174{\cdot}87$ cm $female = 172{\cdot}98$ cm	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • substitution into either formula *High Partial Credit:* • one relevant height found **Note**: Correct answer without work, award full marks
(b)	$2{\cdot}3x + 65{\cdot}53 = 184$ $x = 51{\cdot}51$ cm	**Scale 5B (0, 2, 5)** *Partial Credit:* • equation formed
(c)	$2{\cdot}3(44{\cdot}2) + 65{\cdot}53 = 167{\cdot}19$ $171 - 167{\cdot}19 = 3{\cdot}81$ $\frac{3{\cdot}81}{171} \times 100 = 2{\cdot}228\%$ $2{\cdot}23\%$	**Scale 10D (0, 4, 6, 8, 10)** *Low Partial Credit:* • formula $= 167{\cdot}19$ • correct substitution into formula • correct answer no work *Mid Partial Credit:* • error found, $3{\cdot}81$ *High Partial Credit:* • % error formulated • $97{\cdot}77\%$ found
(d)	$2{\cdot}3x + 65{\cdot}53 = 2{\cdot}5x + 54{\cdot}13$ $0{\cdot}2x = 11{\cdot}4$ $x = 57$ cm --- *m*(57) ***or*** *f*(57) = 196·63 cm	**Scale 10C (0, 4, 6, 10)** *Low Partial Credit:* • Formulas equated *High Partial Credit:* • transpose completed • length of femur found (x) and stops

(e) **(i)**	$P = \frac{72{\cdot}5}{1{\cdot}6^3}$ $P = 17{\cdot}7$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • some relevant substitution into formula *High Partial Credit:* • formula fully substituted
(e) **(ii)**	$Ph^3 = M$ $h = \sqrt[3]{\frac{M}{P}}$	**Scale 10B (0, 5, 10)** *Partial Credit:* • some relevant transpose
(e) **(iii)**	$13 = \frac{67{\cdot}5}{h^3}$ $h = 1{\cdot}7316$ $h = 1{\cdot}73$ **or** $h = \sqrt[3]{\frac{M}{P}}$ $h = \sqrt[3]{\frac{67{\cdot}5}{13}}$ $h = 1{\cdot}73$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • some relevant substitution into formula • correct answer without work *High Partial Credit:* • formula fully substituted

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