Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2019

Mathematics

Paper 2
Higher Level

Monday 10 June
Morning 9:30 to 12:00

300 marks

Examination Number				

Centre Stamp

Running Total	

For Examiner					
Q.	Ex.	Adv. Ex.	Q.	Ex.	Adv. Ex.
1			11		
2			12		
3			13		
4			14		
5					
6					
7					
8					
9					
10			Total		

Grade

Instructions

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.

You may lose marks if you do not include the appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1 **(Suggested maximum time: 5 minutes)**

John measures the heights of eight students in his class. Their heights, in cm, are:

141 165 167 168 169 170 172 172

(a) Work out the **range** of the data, in cm.

(b) Write down which of these heights is an **outlier**. Give a reason for your answer.

Answer: Reason:

(c) Work out the **mean** of the data. Give your answer in cm.

(d) John also measured the heights of the teachers in his school.
John says: "Most of the teachers in my school have a height that is greater than 175 cm."

Put a tick in the correct box to show which of the following statements **must** be true, based on what John says. Tick (✓) **one** box only. Give a reason for your answer.

☐ The **mean** height of the teachers is greater than 175 cm

☐ The **median** height of the teachers is greater than 175 cm

☐ The modal height of the teachers (the **mode**) is greater than 175 cm

Reason:

Question 2 **(Suggested maximum time: 20 minutes)**

The following question was asked as part of a survey:

How do you usually travel to work? Circle **one** of the following.			
Walk or cycle	Bus or train	Car	Other

(a) Put a tick in the correct box below to show what type of data this question would give. Tick (✓) **one** box only.

Numerical Continuous	Numerical Discrete	Categorical Nominal	Categorical Ordinal
☐	☐	☐	☐

The table below gives information about the ways men and women in Ireland travelled to work in 2006 and in 2016.

	Percentage in 2006		**Percentage in 2016**	
Way of travelling	**% Men**	**% Women**	**% Men**	**% Women**
Walk or Cycle	12·1	15·4	12·2	12·6
Bus or Train	8·0	11·3	8·3	10·5
Car	62·6	71·4	60·0	71·9
Other	15·3		14·2	0·4
Not stated		1·2	5·3	4·6
Total	**100·0**	**100·0**	**100·0**	

(b) Write down the **modal** way of travelling (the mode) for both men and women in 2016.

Answer: ☐

(c) Complete the table by filling in the three missing percentages.

(d) Evie says: "In 2006, women were more likely to **walk or cycle** to work than men."
Show that Evie is correct, using numbers from the table.

(e) Alison says: "More women **must have** travelled to work by car in 2016 than in 2006, because $71{\cdot}9\%$ travelled by car in 2016 but only $71{\cdot}4\%$ did in 2006."

Is Alison correct? Tick (✓) **one** box only. Give a reason for your answer.

Alison **is** correct ☐ Alison is **not** correct ☐

Reason:

(f) In total in 2016, roughly $990\,000$ men and $880\,000$ women travelled to work.

Using the figures in the table, work out the **total percentage** of people who travelled to work by **bus or train** in 2016. Give your answer correct to three significant figures.

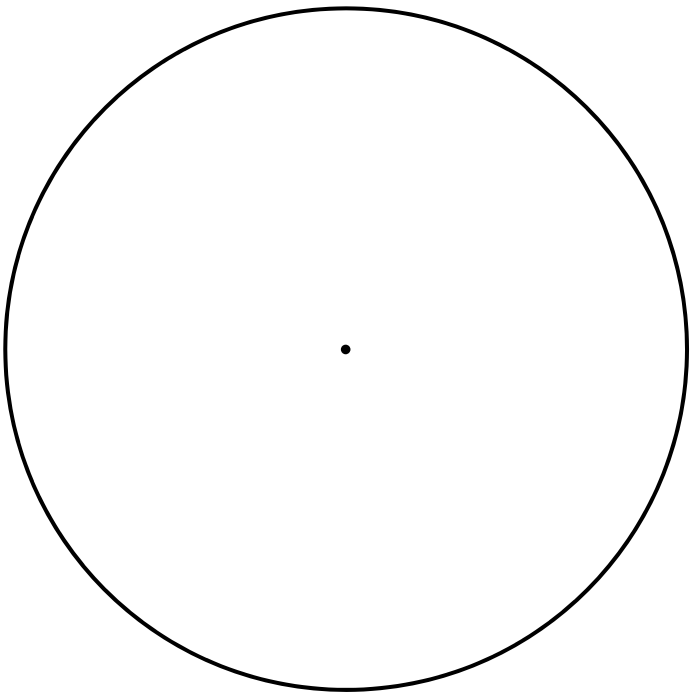
This question continues on the next page.

(g) **(i)** Complete the table below showing the angle in a pie chart for each of the ways men travelled to work in 2016. Show your work. Give each angle correct to the nearest degree.

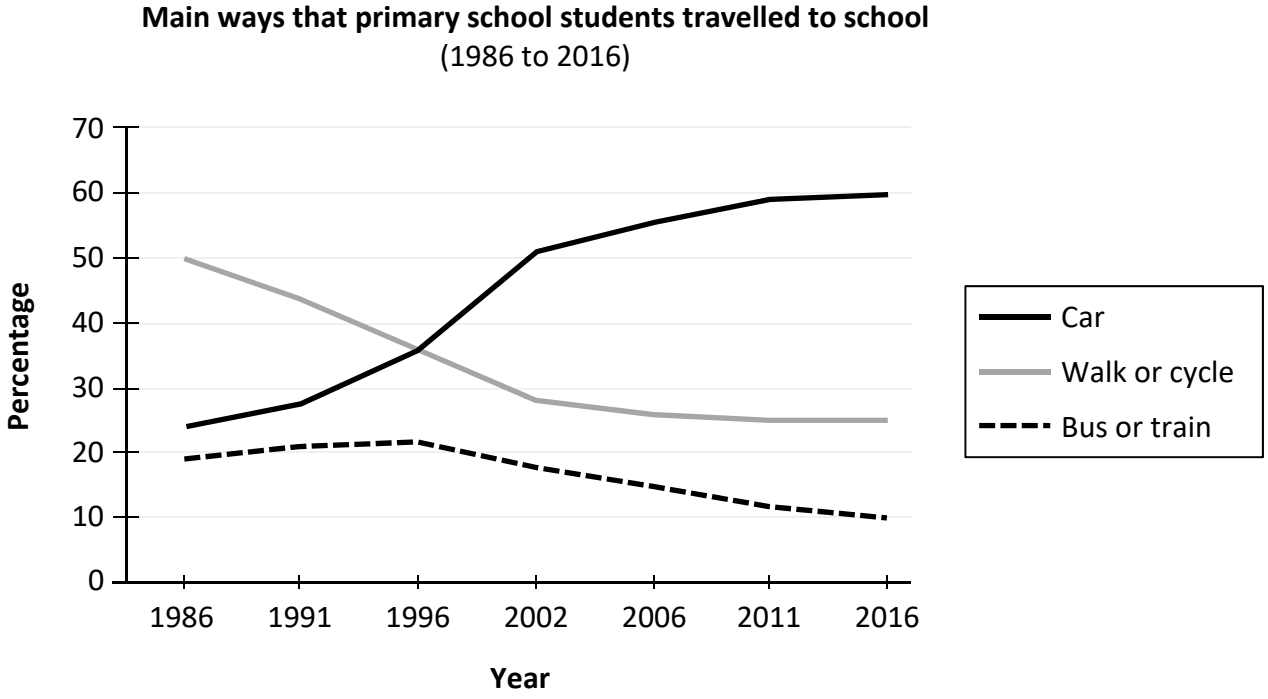
Way of travelling	Walk or cycle	Bus or train	Car	Other	Not Stated
% of men in 2016	12·2	8·3	60·0	14·2	5·3
Angle in pie chart (nearest degree)					

(ii) Complete the pie chart below to show the data in the table above.
Label each sector **and** the size of each angle clearly.

The ways men travelled to work in 2016



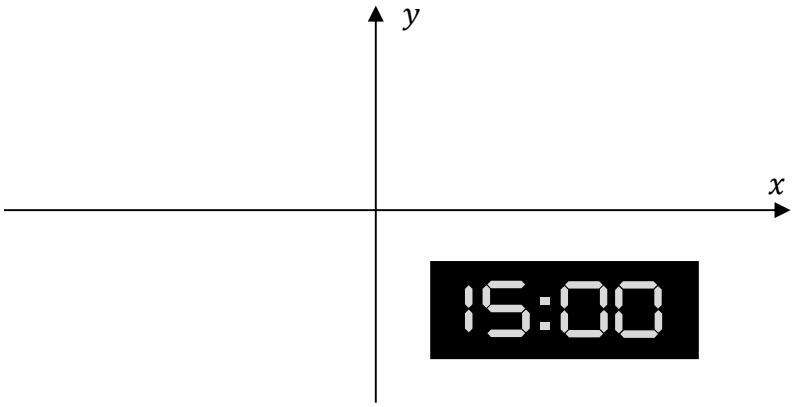
The diagram below shows the main ways that primary school students travelled to school from 1986 to 2016.



(h) Write a paragraph to describe the information the diagram gives about the ways that primary school students travelled to school over the period from 1986 to 2016.
In your answer, you should use the information in each of the three lines in the diagram (——— , ———, and - - - -).

Question 3 **(Suggested maximum time: 5 minutes)**

The co-ordinate diagram below shows the screen of a digital clock at 15:00.



The images P, Q, S, and T of this screen under four transformations are as follows:

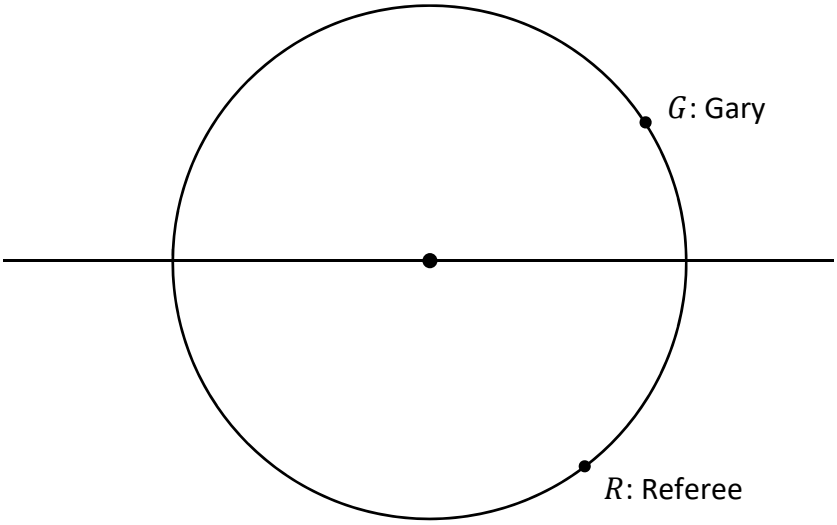
Image P	Image Q	Image S	Image T
15:00	00:51	12:00	00:21

Complete the table below to match each image to the transformation that produced it.
Use each letter (P, Q, S, and T) only once.

Transformation	Image (P, Q, S, or T)
Axial symmetry in the x-axis	
Axial symmetry in the y-axis	
Translation	
Central symmetry in the origin $(0, 0)$	

Question 4 **(Suggested maximum time: 5 minutes)**

The diagram below shows the centre circle of a soccer pitch. The centre of the circle is marked. At the start of a game, Gary and the referee stand at two points on the circle, G and R, as shown.



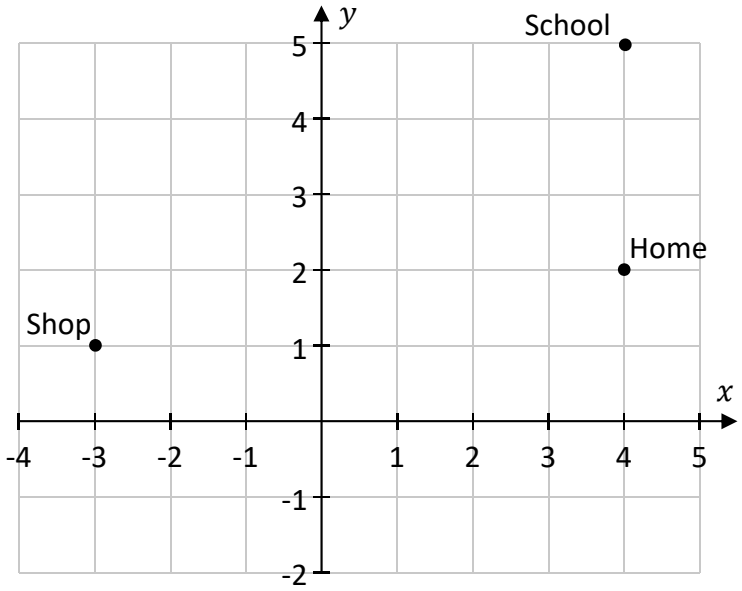
Jorge stands at the point J **on the circle** so that the angle at the referee is $90°$, that is $|\angle JRG| = 90°$.

(a) Using only a straight edge, **construct** the point J on the diagram above. **Label** the point J.

(b) State a theorem or corollary that justifies your construction in part **(a)**.

Question 5 **(Suggested maximum time: 20 minutes)**

The co-ordinate diagram below shows part of the town where Cliodhna lives.
Each small square in the grid has sides of length 1 cm.



(a) Write down the co-ordinates of the Shop, Home, and School.

Shop: (,) Home: (,) School: (,)

(b) The Library is at the midpoint between Home and the Shop.
Write down the co-ordinates of the Library.

(c) Work out the **distance** from Home to the Shop on the diagram.
Give your answer in cm, correct to two decimal places. Show your working out.

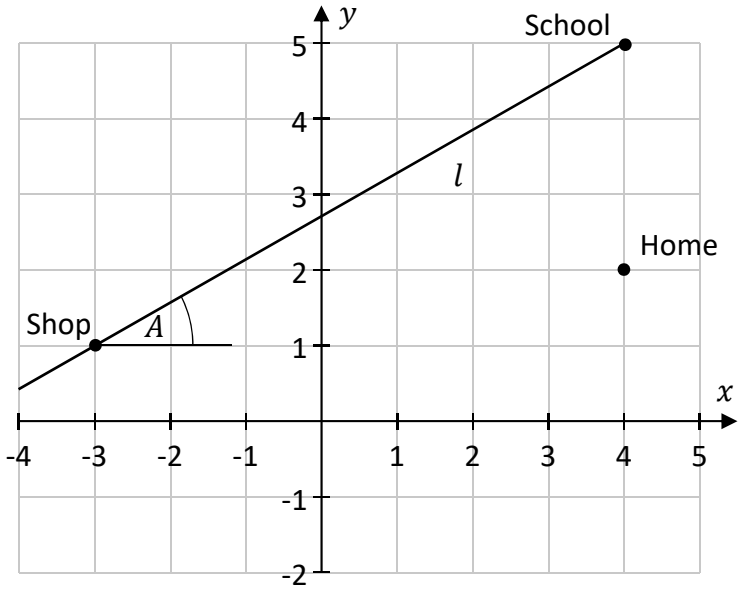
(d) The distance on the diagram from the Shop to the School is roughly $\mathbf{8{\cdot}1}$ **cm**.
The diagram is to a scale of $\mathbf{1:2500}$.

Work out the **actual distance** from the Shop to the School. Give your answer in metres.

(e) Cliodhna walks from the Shop to the School.
Explain why the distance she walks is probably **more** than the actual distance from part **(d)**.

This question continues on the next page.

The co-ordinate diagram below has the line l drawn from the Shop to the School.
The angle A is marked.



(f) Work out the slope of the line l. Give your answer as a fraction.

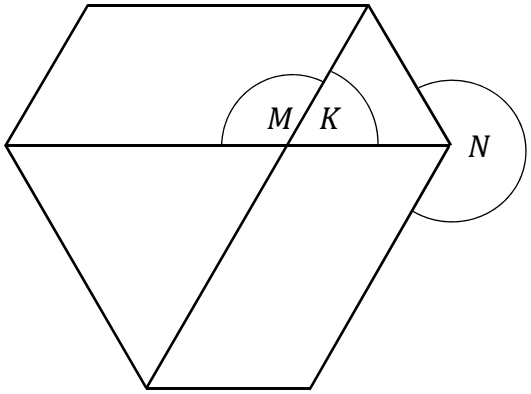
(g) Find the **equation** of the line l.
Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

(h) Use trigonometry and the slope of l to work out the **size** of the angle A.
Give your answer in degrees, correct to one decimal place.

Answer:

Question 6 **(Suggested maximum time: 10 minutes)**

The diagram below is made up of two equilateral triangles and two parallelograms.
Three angles are marked.



(a) Write the correct letter in each space in the table below to show what type of angle each one is. Use each letter only once.

Type of angle	Acute	Reflex	Obtuse
Angle (K, M, or N)			

(b) Work out the size of each of the angles K, M, and N.
Remember that the two triangles are equilateral.

$|\angle K| =$ $|\angle M| =$ $|\angle N|$

(c) The two equilateral triangles are **not** the same size as each other.
Siobhán says: "The two triangles are similar, but they are **not** congruent."

(i) Give a reason why the two triangles are **similar**.

(ii) Give a reason why the two triangles are **not congruent**.

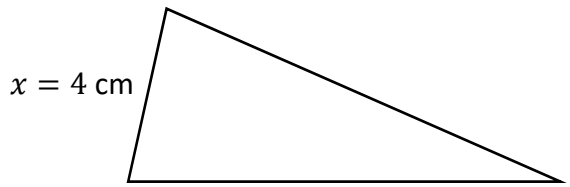
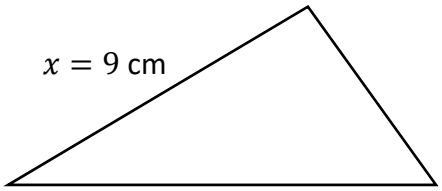
(d) The small triangle in the diagram has sides of length 4 cm.
The big triangle has sides of length 8 cm.

Use this information to **construct** the diagram in the space below.
Show clearly any construction lines used.

Question 7 **(Suggested maximum time: 15 minutes)**

A triangle has one side of length 10 cm and another side of length x cm.
The **perimeter** of this triangle is $\mathbf{26}$ **cm** in length.

(a) Fill in the length of the third side in each case in the table below.

Diagram A	**Diagram B**
	
Perimeter $= 26$ cm Length of third side $=$ _____ cm	Perimeter $= 26$ cm Length of third side $=$ _____ cm

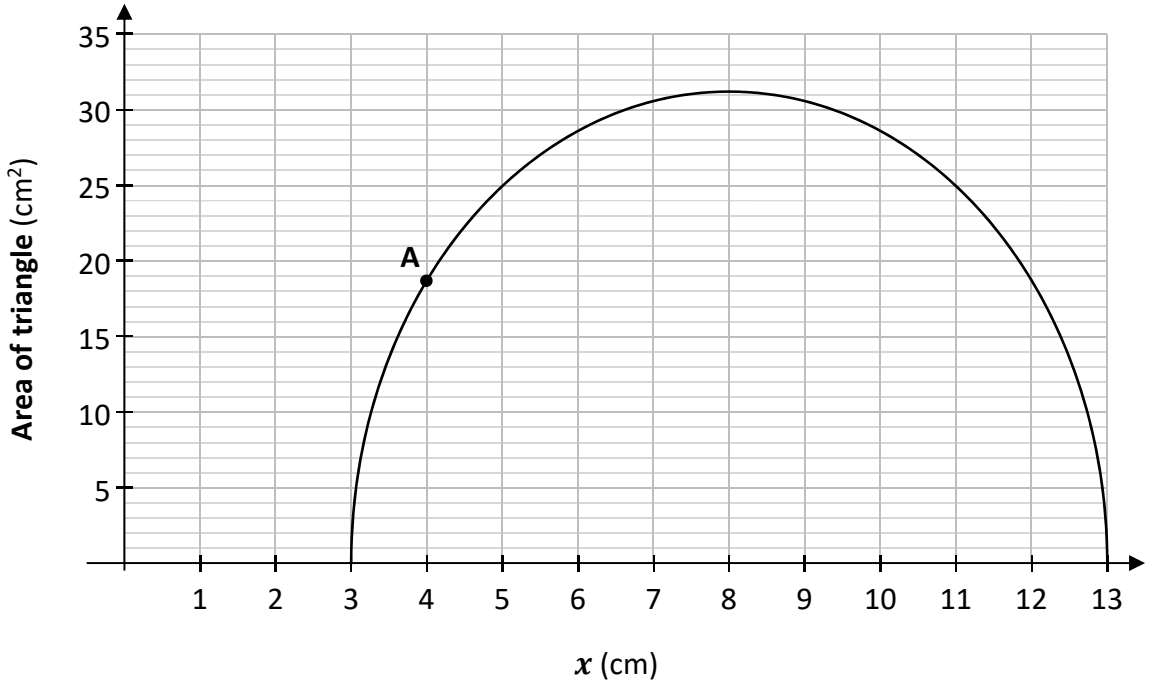
(b) Find the three values of x that make the triangle an **isosceles** triangle.

Answer: $x =$ ☐ cm ,

$x =$ ☐ cm,

or $x =$ ☐ cm.

(c) The graph below shows the **area** of the triangle for all values of x between 3 and 13. The point **A** is marked.



(i) **A** is the point on the graph with $x = 4$, so it represents the triangle in **Diagram A**. Use the point **A** on the graph to estimate the **area** of the triangle in **Diagram A**.

Answer: _____ cm^2

(ii) **Plot** the point **B** on the graph to represent the triangle in **Diagram B**.
Label the point **B**.

(iii) The graph is symmetrical.

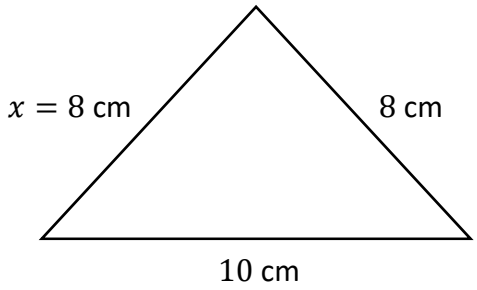
Draw the axis of symmetry of the graph on the diagram above.
Write down the **equation** of the axis of symmetry.

Equation: _____

This question continues on the next page.

(d) When $x = 5$ cm, the triangle has sides of length 10 cm, 5 cm, and 11 cm.
Show that this is **not** a right-angled triangle.

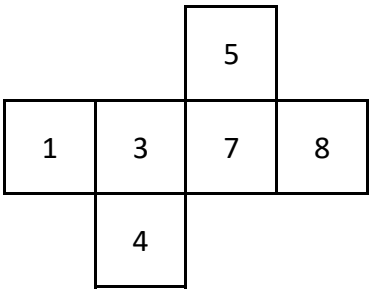
(e) The diagram below shows the triangle with the biggest area.



Work out the **area** of this triangle. Give your answer in the form $5\sqrt{n}$ cm^2, where $n \in \mathbb{N}$.

Question 8 **(Suggested maximum time: 5 minutes)**

The net below is folded to make a fair die.



(a) What number will be opposite the number 7 on the die? Answer:

(b) The die is rolled once. Find the probability that the number rolled is greater than 6.

(c) The die is rolled 60 times.
Work out how many times you would expect an odd number to be rolled.

(d) The following three terms are used when discussing experiments in probability:

Outcome **Sample space** **Event**

Write each of these terms in the table below to match each term to its description.

Description	Term
The set of all the possible outcomes of an experiment.	
One possible result of an experiment.	
A subset of the sample space – a collection of one or more outcomes.	

Question 9 **(Suggested maximum time: 15 minutes)**

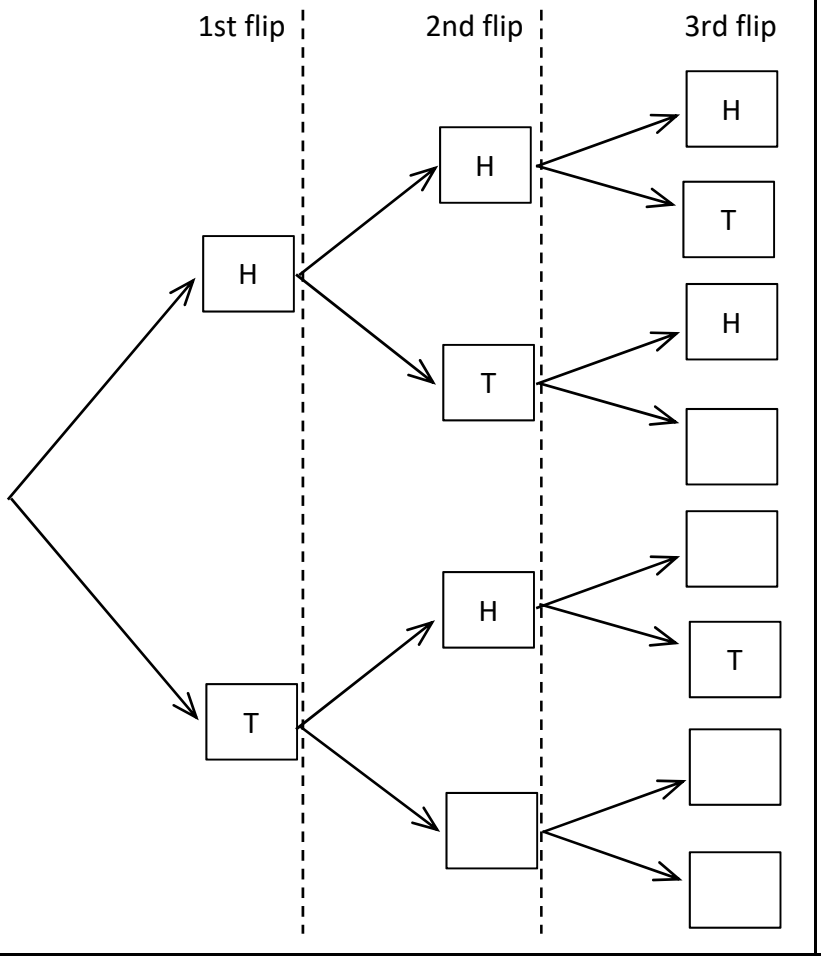
Ger flips a coin 3 times. Each time, he can get heads (**H**) or tails (**T**).
The diagram in the table below shows how he can get each of the 8 different outcomes.

(a) **Complete** the table below by doing **each** of the following:

(i) Write in each of the missing letters (H or T) in the **diagram**

(ii) Write in the five missing **outcomes**

(iii) Write in the **number of heads (H)** in each of the outcomes.

Diagram	Outcome	Number of Heads (H)
	HHH HTH THH 	3 2

(b) Each outcome is equally likely. Fill in the table below to show the **probability** that Ger gets each of the given number of heads.

Number of Heads (H)	0	1	2	3
Probability that Ger gets this number of heads				

Fidelma flips a coin 8 times. Each time, she can get heads (**H**) or tails (**T**).

(c) **(i)** Write down the number of outcomes that have **no heads**.

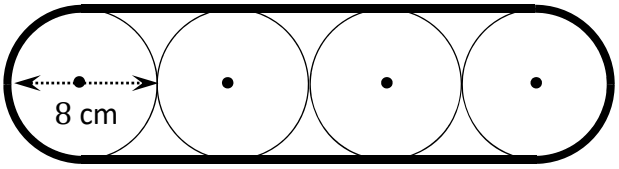
(ii) Work out the number of outcomes that have exactly **1 head**.

(d) Work out the **total** number of outcomes when Fidelma flips the coin 8 times.

Question 10 **(Suggested maximum time: 10 minutes)**

(a) Work out the circumference of a circle with a **diameter** of 8 cm.
Give your answer correct to one decimal place.

The rubber track for a toy digger goes around four circular wheels of diameter 8 cm, as shown.



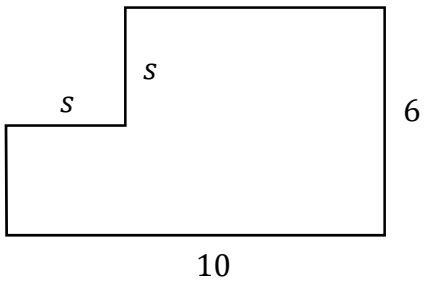
(b) Calculate the length of the rubber track that goes around the four wheels.
Give your answer correct to one decimal place.

Every time the wheels turn fully, the digger travels a distance equal to one wheel's circumference.

(c) Work out how many times each wheel will turn **fully** when the digger travels a distance equal to the length of its rubber track.

Question 11 **(Suggested maximum time: 10 minutes)**

In the diagram below, two of the sides have a length of s, where $0 < s < 6$ and $s \in \mathbb{R}$.
All angles are $90°$ or $270°$.



(a) Find a formula (in algebra) for the **area** of this shape, in terms of s.

(b) Show that (or explain why) the **perimeter** of this shape is always 32, no matter what the value of s.

Question 12 **(Suggested maximum time: 5 minutes)**

Prove that each exterior angle of a triangle is equal to the sum of the interior opposite angles.

Diagram:

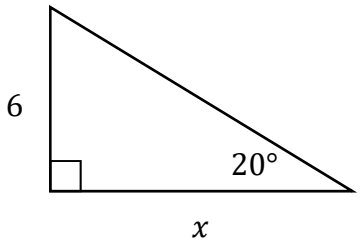
Given:

To Prove:

Proof:

Question 13 **(Suggested maximum time: 10 minutes)**

(a) Use trigonometry to work out the value of x in the diagram below.
Give your answer correct to two decimal places.

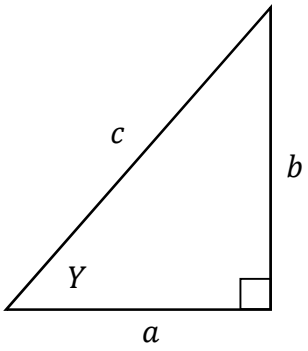


(b) A different right-angled triangle is shown on the right.
It has sides of length a, b, and c.
One of the angles is Y.

In this triangle, $\boldsymbol{a + b > c}$.

Use this to prove that:

$$\cos Y + \sin Y > 1$$



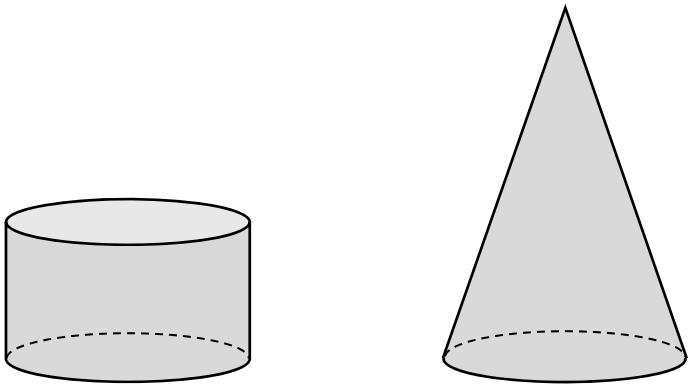
Question 14 **(Suggested maximum time: 5 minutes)**

A cylinder has a radius of r cm and a height of 10 cm.

This cylinder is melted down and made into a cone.
The cone also has a radius of r cm, and it has a height of h cm.

10% of the volume of the cylinder is lost during this process.

Work out the **percentage increase** in the height of the cylinder when it is made into the cone.



Page for extra work.
Label any extra work clearly with the question number and part.

Junior Certificate – Higher Level

Mathematics – Paper 2

Monday 10 June
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