Question 1

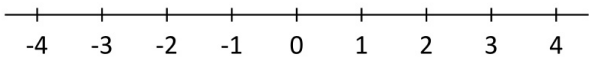
Question 3 **(30 marks)**

(a) Solve the following equation in $x \in \mathbb{R}$:

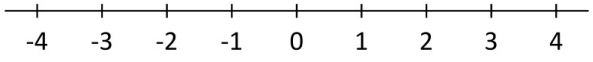
$$3(2x + 4) - 5 = 3$$

(b) Graph each of the following inequalities on the given number line.
Note that x comes from a **different set of numbers** in each case.

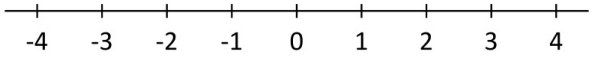
(i) $x \leq 2{\cdot}8$, where $x \in \mathbb{R}$.



(ii) $x \leq 2{\cdot}8$, where $x \in \mathbb{Z}$.



(iii) $x \leq 2{\cdot}8$, where $x \in \mathbb{N}$.



(c) Use algebra to solve the following simultaneous equations:

$$\begin{aligned} x - y + 1 &= 0 \\ 2x + 4y &= 19 \end{aligned}$$

Question 2

Question 5 **(30 marks)**

(a) A company is repairing a railway line. As part of this work, it is laying new railway tracks.

(i) It costs €12 000 to lay 240 metres of railway track.
Work out how much would it cost to lay 320 metres of railway track, at the same rate.

(ii) It would take 6 people 8 days to replace the tracks on a different part of the railway line. Work out how long it would take 4 people to replace these tracks, each working at the same speed.

(b) Two towns, **A** and **B**, are 120 km apart.
A train travels from **A** to **B** at an average speed of 180 km/hour.

(i) Work out the time it takes the train to get from **A** to **B**.
Give your answer in hours, as a fraction.

(ii) On the way back from **B** to **A**, the train travels at an average speed of 220 km/hour.
Work out the average speed of the train for the entire journey, while it is travelling from **A** to **B** and back again. Give your answer in km/hour.

Hint: first work out the total time taken for the entire journey.

Question 3

Question 4 **(30 marks)**

(a) $g(x) = x^3 - 7x^2 + x - 12$, where $x \in \mathbb{R}$.

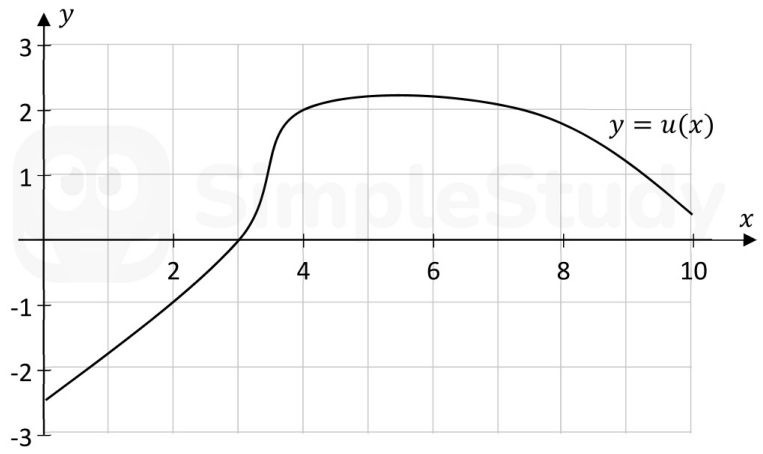
(i) Work out the value of $g(5)$.

(ii) Find $g'(x)$, the derivative of $g(x)$.

(iii) $g'(5) = 6$.

Use this to find the equation of the **tangent** to the curve $y = g(x)$ when $x = 5$.
Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{R}$.

(b) The graph of the function $y = u(x)$ is shown below, for $0 \leq x \leq 10$, $x \in \mathbb{R}$.



$u'(x)$ is the derivative of $u(x)$.

(i) Using the graph, write down a value of x for which $u'(x)$ is **negative**.

$x =$

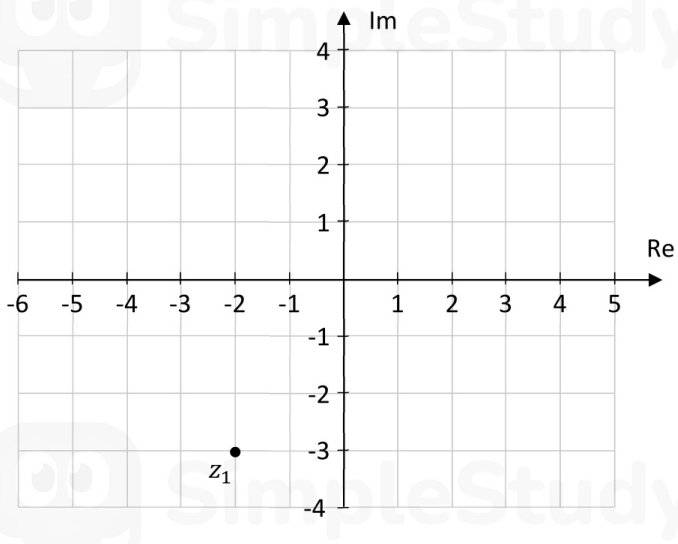
(ii) On the diagram above, draw the tangent to $u(x)$ at the point $(4, 2)$ **and** use the tangent that you draw to work out an estimate for the value of $u'(4)$.

$u'(4) =$ _____ , approximately

Question 4

Question 1 **(30 marks)**

The complex number z_1 is shown on the Argand diagram below.



(a) Using the Argand diagram:

(i) write down the values of z_1 and $\overline{z_1}$, where $\overline{z_1}$ is the complex conjugate of z_1

$z_1 =$ $\overline{z_1} =$

(ii) plot and label $\overline{z_1}$ on the Argand diagram above.

z_2 and z_3 are two other complex numbers.

$z_2 = -5 + 3i$ and $z_3 = 4 - 2i$, where $i^2 = -1$.

(b) Plot and label z_2 and z_3 on the Argand diagram on the previous page.

(c) Write $z_2 - z_3$ in the form $a + bi$, where $a, b \in \mathbb{R}$, $i^2 = -1$, **and** hence find $|z_2 - z_3|$.

$z_2 - z_3 =$ _____

$|z_2 - z_3| =$ _____

(d) Investigate if $z_3 = 4 - 2i$ is a solution of the equation $z^2 + 2iz - 7i = 0$.

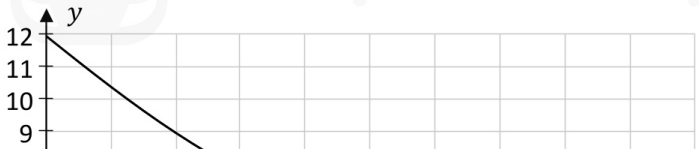
Conclusion: _____

Question 5

Question 6 **(30 marks)**

The graph of a function $k(x)$ is shown on the co-ordinate diagram below, for $0 \leq x \leq 5,\ x \in \mathbb{R}$.

(a) Use the graph to answer **parts (a)(i)** and **(a)(ii)**.





(i) Estimate the value of $k(x)$ when $x = 2{\cdot}5$.
Show your work on the graph.

(ii) Estimate the range of values of x, for which $k(x) \leq 7$.
Show your work on the graph.

(b) Liam uses the **Trapezoidal Rule** to estimate the area under the graph of $k(x)$ between 0 and 5.

He takes the following measurements from the graph on the previous page:

x	0	1	2	3	4	5
$k(x)$	12	9	6·8	5·1	3·8	2·8

(i) Using the values of $k(x)$ in the table above, show that the Trapezoidal Rule will give an estimate of $32{\cdot}1$ units^2 for this area.

(ii) The actual area under $k(x)$, between 0 and 5, is $31{\cdot}8$ units^2. Liam estimated the area was $32{\cdot}1$ units^2 by using the Trapezoidal Rule.

Work out the **percentage error** in Liam's estimate.
Give your answer as a percent, correct to 2 decimal places.

Question 6

Question 7 **(50 marks)**

(a) During an experiment, the number of bacteria in a dish can be modelled by the function:

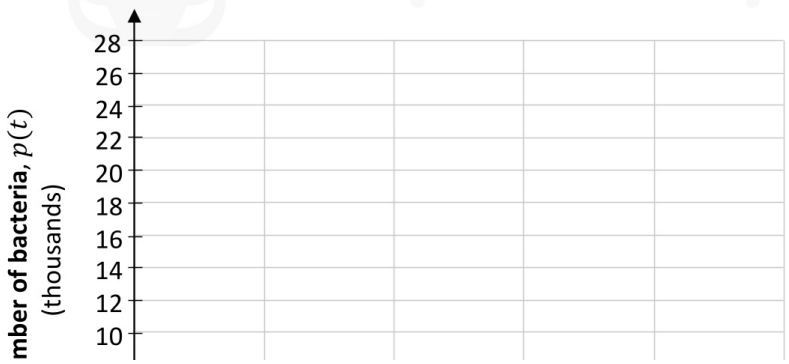
$$p(t) = t^3 - 6t^2 + 6t + 20$$

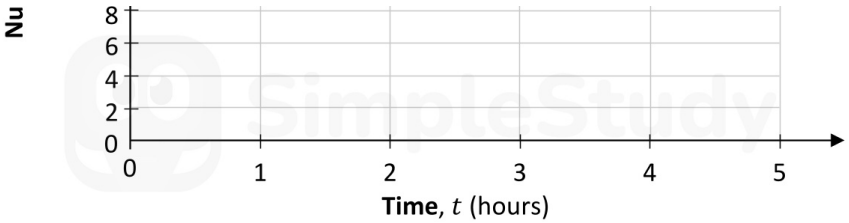
where $p(t)$ is the number of bacteria, in thousands, and t is the time in hours since the start of the experiment, with $0 \leq t \leq 5$, $t \in \mathbb{R}$.

(i) Complete the table below to show the values of $p(t)$ for the given values of t.

Time, t (hours)	0	1	2	3	4	5
Number of bacteria, $p(t)$ (thousands)			16		12	

(ii) Draw the graph of $p(t)$ on the axes below, for $0 \leq t \leq 5$, $t \in \mathbb{R}$.





(iii) Find $p'(t)$, the derivative of $p(t)$.
Remember that:

$$p(t) = t^3 - 6t^2 + 6t + 20$$

(iv) Find the value of $p'(2)$.

(v) $p'(4) = 6$.

Explain what this means in terms of the number of bacteria.

This question continues on the next page.

(b) The number of bacteria in a **different** experiment can be modelled by the function:

$$k(t) = 3000 \times 2{\cdot}72^{0{\cdot}5t}$$

where $k(t)$ is the number of bacteria and t is the time in hours since the start of the experiment, with $0 \leq t \leq 8$, $t \in \mathbb{R}$.

There are 3000 bacteria at the beginning of the experiment.

(i) Use $k(t)$ to find the number of bacteria after 1 hour **and** after 2 hours.
Give each answer correct to the nearest whole number.

After 1 hour

After 2 hours

(ii) After n hours, there are at least $35\,000$ bacteria according to $k(t)$, where $n \in \mathbb{N}$.
By continuing the pattern above, or otherwise, find the smallest possible value of n.

Question 7

Question 7 **(50 marks)**

(a) When it rains on land, some of the rain soaks into the land, and the rest runs off the land. The runoff curve number, C, is a number used when estimating the amount of rain that runs off a particular area of land. C is given by:

$$C = \frac{1000}{S + 10}$$

where S is a measure of the maximum amount of rain that can soak into the soil.

(i) Find the value of C when $S = 15$.

(ii) When S **increases**, does C increase or decrease?

Justify your answer, using the equation $C = \frac{1000}{S+10}$.

Answer (increases **or** decreases):

Justification:

(iii) Rearrange the equation to write S in terms of C.

(iv) The values for C go from 30 to 100.
Use your answer from **part (a)(iii)**, or otherwise, to find the range of values of S.

Answer: S goes from _____ to _____

(b) Over the course of one day (24 hours), the probability that it is raining in Waterville, $P(t)$, could be modelled by the following function:

$$P(t) = 0{\cdot}3 + 0{\cdot}02\,t.$$

Here, $P(t)$ is the probability that it is raining t hours after the start of the day, with $0 \leq t \leq 24$, $t \in \mathbb{R}$.

(i) Write down the probability that it is raining at the start and the end of the day, that is, find the value of $P(0)$ and $P(24)$.

(ii) Find the value of $P'(t)$, the derivative of t, **and** explain what the value of $P'(t)$ means in this context.

$P'(t)$:

Meaning of $P'(t)$:

(iii) At a certain time, the probability that it is raining, according to this model, is $0{\cdot}35$. Work out the probability that it is raining exactly 1 hour later, according to this model.

Question 8

Question 7 **(50 marks)**

The rates and thresholds of the Universal Social Charge (USC) in Ireland (excluding the top rate) during 2020 are given in the table below.

Standard rates and thresholds for USC 2020

Annual Income	Rate

First €12 012	0·5%
Next €8472	2%
Next €49 560	4·5%
Balance	(Top Rate)

Source: *www.revenue.ie*

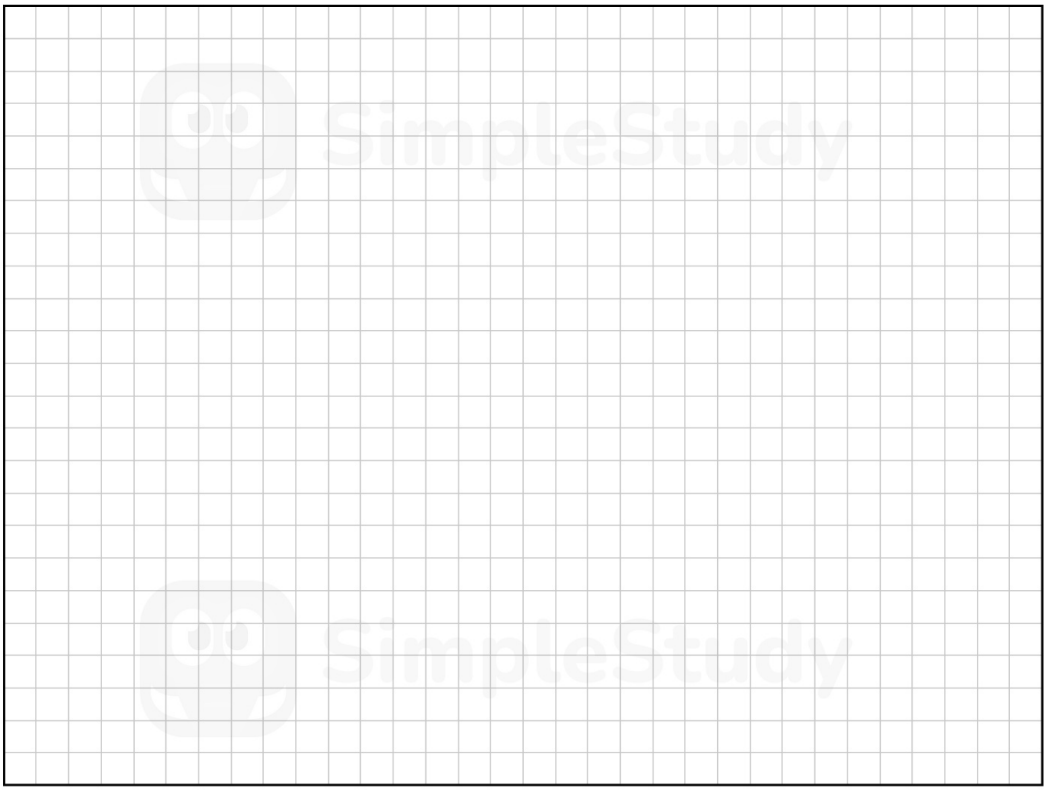
(a) At what level of annual income does a worker start paying the top rate of USC?

(b) How much USC will a worker have paid in total if they pay the maximum amount due at each of the first three rates?

(c) John's annual income is €54 800. Find the amount of USC he will pay.

(d) Mary pays €1602·72 in USC in 2020. Find her annual income.

(e) Patrick has an annual income of €83 000 and pays a total of €3496·18 in USC.
Find the top rate of USC.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