Question 4 **(30 marks)**

(a) The circle c has equation:

$$(x-h)^2+(y+3)^2=12$$

where $h \in \mathbb{R}$.

(i) Write down the centre and radius of the circle c.
Give your answer in terms of h, where appropriate.

Centre = (,) Radius = _____

(ii) The perpendicular distance from the line $x-4y+7=0$ to the centre of the circle c is 5 units.

Work out the two possible values of h. Give each answer in surd form.

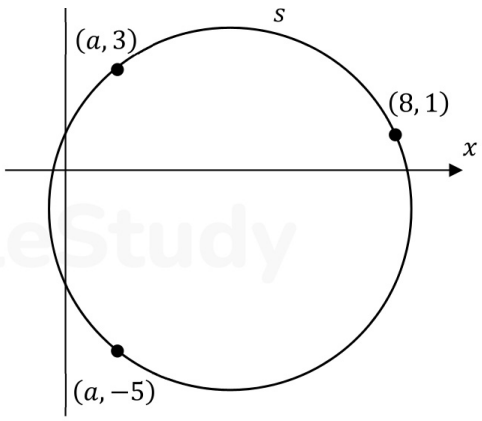
(b) The circle s passes through the points



$(8, 1)$, $(a, 3)$, and $(a, -5)$, as shown in the diagram on the right (not to scale), where $0 < a < 5,\ a \in \mathbb{R}$.

The radius of the circle s is $\sqrt{20}$.

Find the equation of the circle s.



Question 2

Question 6 **(30 marks)**

(a) State whether the following statement is true or false:

Two angles are vertically opposite if, and only if, they are equal in size.

Justify your answer.

Answer (true or false):

Justification:

(b) The two diagrams below show the same rectangle, $ACEG$ (not to scale).
The points B, D, and F lie on $[AC]$, $[CE]$, and $[EG]$, respectively, as shown.
$|AB| = 20$ cm, $|BC| = 30$ cm, and $|AG| = 90$ cm.
$|\angle GFA| = |\angle EFD| = |\angle DBC| = \theta$, where $\theta \in \mathbb{R}$.

In **Diagram B** below, $[GE]$ and $[BD]$ are extended, and they meet at the point H.

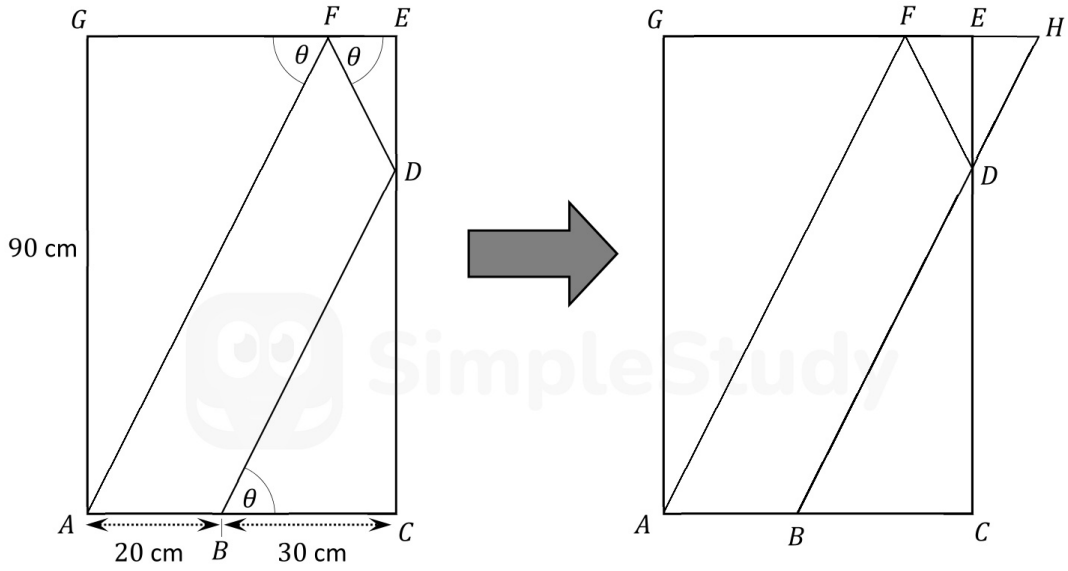


Diagram A **Diagram B**

(i) Prove that $|FE| = |EH|$, in **Diagram B**. Use congruent triangles.
Give a reason for each statement that you make in your proof.

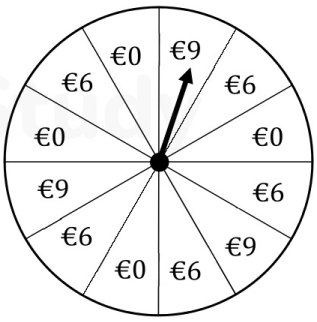
(ii) Hence, or otherwise, find the size of the angle θ.
Give your answer correct to the nearest degree.

Question 3

Question 1 **(30 marks)**

A circular spinner has 12 sectors, as follows:

- 5 sectors are labelled €6
- 3 sectors are labelled €9
- The rest are labelled €0.



In a game, the spinner is spun once.
The spinner is equally likely to land on each sector.
The player gets the amount of money shown on the sector that the spinner lands on.

(a) Fiona plays the game a number of times.

Work out the probability that Fiona gets €6, then €9, then €6 the first three times she plays.
Give your answer correct to 4 decimal places.

(b) Rohan also plays the game a number of times.

Find the probability that Rohan gets €9 for the 3rd time, on the 8th time that he plays the game. Give your answer correct to 4 decimal places.

(c) Olga plays the game 2 times.

Find the probability that Olga gets less than €16 in **total** from playing the game.
Give your answer correct to 4 decimal places.

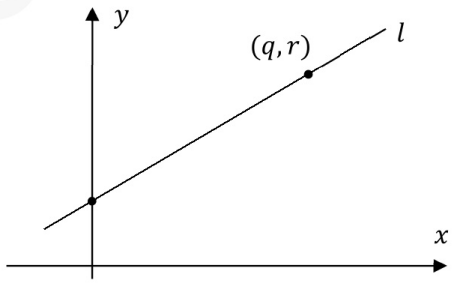
Question 4

Question 2 **(30 marks)**

(a) The points $A\ (8, -4)$ and $B\ (-1, 3)$ are the endpoints of the line segment $[AB]$.

Find the coordinates of the point C, which divides $[AB]$ internally in the ratio $4 : 1$.

(b) The line l has a slope of m and contains the point (q, r), where $m, q, r \in \mathbb{R}$ are all positive.
Find the co-ordinates of the point where l cuts the y-axis, in terms of m, q, and r.



Answer: (,)

(c) The line k has a slope of -2.
The line j makes an angle of $30°$ with k.

Find **one** possible value of the slope of the line j.
Give your answer in the form $d + e\sqrt{f}$, where $d, e, f \in \mathbb{Z}$.

Question 5

Question 1 **(30 marks)**

(a) The table below gives some details on the number of different types of student in a university. There are 22 714 students in the university in total.

	Age (years)		**Total**
	23 or younger	**24 or older**	
Undergraduate	12 785	2922	**15 707**
Postgraduate	1353		
Total		**8576**	**22 714**

(i) Fill in the three missing values to complete the table above.

(ii) One student is picked at random from the students in the university.
Let **O** be the event that the student is 24 years old, or older.
Let **U** be the event that the student is an undergraduate.

Are the events **O** and **U** independent? Justify your answer.

Conclusion: _____

(b) Three people are picked at random from a class.
Find the probability that all three were born on the same day of the week.
Assume that the probability of being born on each day is the same.

(c) There are b boys and g girls in a class, where $b, g \in \mathbb{N}$.
$\frac{3}{5}$ of the students in the class are girls.

4 boys and 4 girls join the class.
One student is then picked at random from the whole class.
The probability that this student is a girl is now $\frac{4}{7}$.

Find the value of b and the value of g.

$b =$ _____ $g =$ _____

Question 6

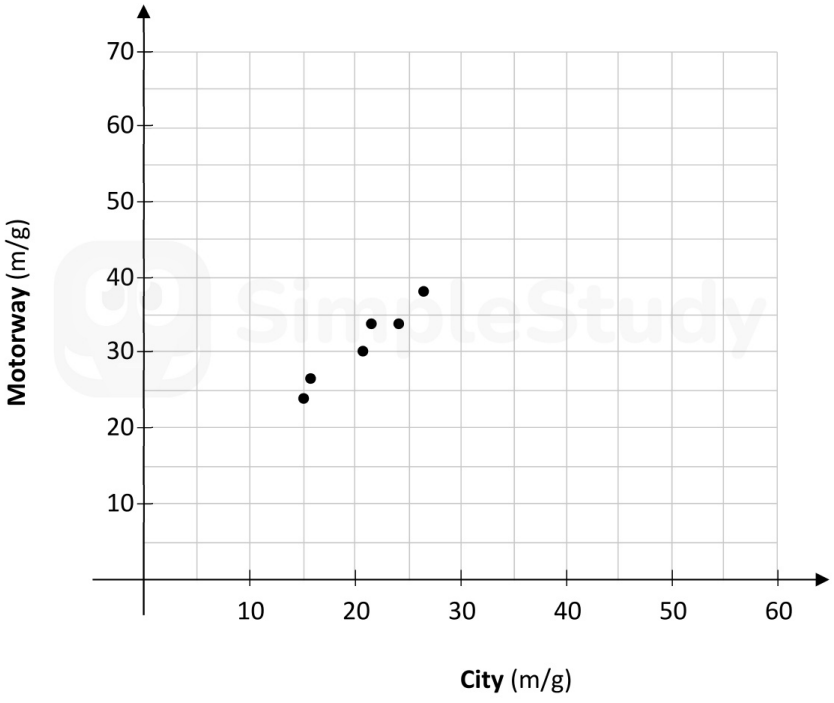
Question 8 **(50 marks)**

(a) Jena is researching fuel consumption in cars. She finds the following data for the number of miles per gallon (m/g) for eight different cars, labelled **A** to **H**, when driving in the city and on the motorway:

Miles per gallon data for city and motorway		
Car	**City** (m/g)	**Motorway** (m/g)
A	22	34
B	27	38
C	24	34
D	16	27
E	15	24
F	21	30
G	**30**	**40**
H	**17**	**30**

(i) The scatterplot below shows this data for cars **A** to **F**.

Using the data in the table above, **plot** and **label** points to represent cars **G** and **H** on the scatterplot below.



(ii) On the scatterplot, **draw** the line of best fit for the data, by eye.

(iii) Two other cars, **K** and **L**, have the miles per gallon values given in the following table.

Use your line of best fit on the scatterplot to fill in an estimate for each of the two missing values in the table below. Show your work on the scatterplot.

Car	**City** (m/g)	**Motorway** (m/g)
K	20	
L		60

(iv) Based on the data given, would you be more confident in the value you estimated for **K** or for **L**? Give a reason for your answer.

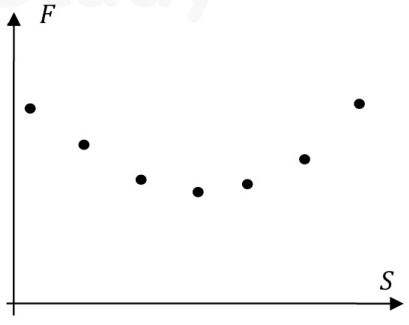
I would be more confident in my value for: **K** ☐ **L** ☐

(Tick (✓) **one** box only)

Reason:

(v) Find the value of r, the correlation coefficient between city and motorway miles per gallon. Use only the values for the 8 cars **A** to **H** in the table on the previous page. Give your answer correct to 3 decimal places.

(b) The scatterplot on the right shows some values of fuel consumption (F) for the given values of engine speed (S), for a particular car.
For the points in this scatterplot, F can be closely approximated by a quadratic function of S.



r_{FS} is the correlation coefficient between F and S, based on the points in this scatterplot.
Give a reason why you might think that r_{FS} is very close to 0.

(c) 13 customers rated their experience in a garage, by giving a whole-number score out of 100. The mean score was 52. The median score was 54. No two scores were the same.

The table below shows the score for each of the 13 customers (in no particular order).
Stephen gave a score of **S** and Mary gave a score of **M**, where **S**, **M** $\in \mathbb{N}$.
Find the least value **and** the greatest value that **S** could be.

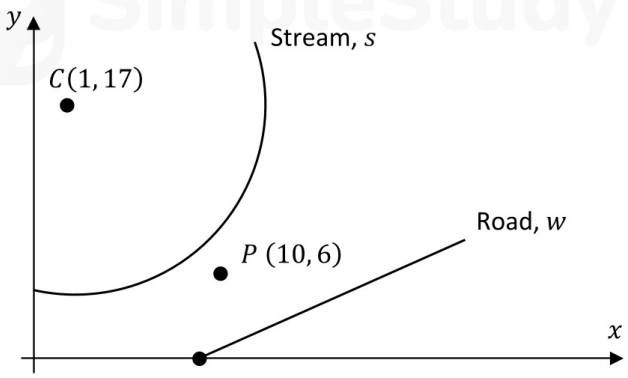
46	68	24	74	42	30	61	54	28	50	57	**S**	**M**

Least value of **S**: _____ Greatest value of **S**: _____

Question 7

Question 9 **(50 marks)**

Ameena, Petro, and Fiadh are taking part in an adventure race.
The co-ordinate diagram below shows part of the course for this race.
Each unit on the diagram represents 100 metres.



(a) The arc in the diagram represents part of a stream.
The arc is part of a circle s with centre $C(1, 17)$ and radius 12.

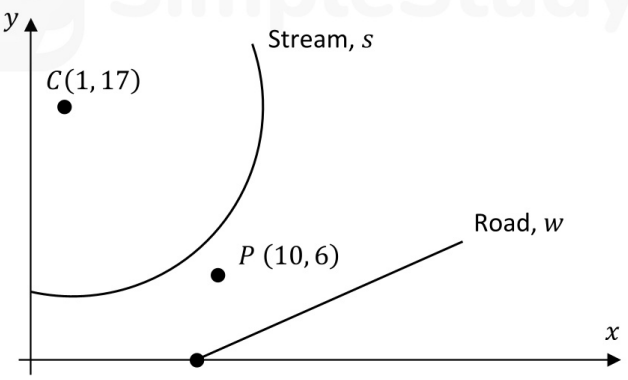
(i) Write down the equation of the circle s.

(ii) Ameena is at the point $(a, 8)$ on the stream (circle s), where $a \in \mathbb{R}$, $a > 0$.
Work out the value of a. Give your answer in surd form.

(iii) Petro is at the point $P(10, 6)$.
Work out the shortest distance from the point P to the stream (circle s).

Give your answer correct to the nearest metre.
Remember that each unit on the diagram represents 100 metres.

The original diagram is shown again below.



(b) There is a straight path, l, that is not shown on the diagram.
l is parallel to the y-axis, and is a **tangent** to the stream s in the first quadrant.

Write down the equation of this path l. Remember that the radius of s is 12.

The line segment w represents a road, where w has the equation:

$$x - 3y = 9$$

for $0 \leq y \leq 8$.

(c) Find the co-ordinates of the point on the road w that is closest to the point $P(10, 6)$.
It might be useful to find the equation of the line through P that is perpendicular to w.

There is more space for work on the next page.

(d) Fiadh is at the point $F(9, 0)$ on the road w.

She travels 1200 m along the road w away from the point F, in the first quadrant, and then stops.

Work out the co-ordinates of the point at which she stops. Give each value correct to 1 decimal place. Remember that each unit on the diagram represents 100 metres.

Question 8

Question 10 **(50 marks)**

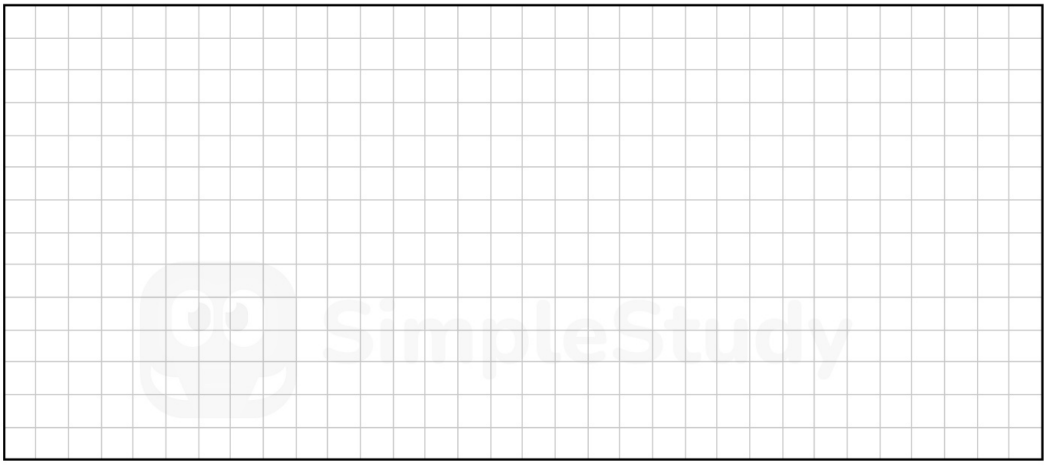
People with O-negative blood type are called "universal donors" because their blood can be given to anyone else. In Ireland approximately 8% of the population have O-negative blood type (source: Blood Transfusion Service).

(a) **(i)** At a blood donation clinic, ten donors give blood, one after the other.
Find the probability that the tenth person is the third O-negative donor.
Give your answer correct to four decimal places.

(ii) At a blood donation clinic, five donors give blood.
What is the probability that at least one of the five donates O-negative blood?
Give your answer correct to four decimal places.

(iii) Find the minimum number of blood donors required, so that the probability that at least one of them is type O-negative is greater than $0{\cdot}97$.

(b) A homeowner has a problem with the heating system in her house. A plumber has identified the problem as a faulty part. The homeowner knows that in 80% of cases a repair of the part will fix the problem and this repair will cost €70. If the repair does not work then a new part will have to be bought costing €150 and there will be an additional labour cost of €80 to replace the old part with the new.
Find the expected value of the cost of fixing this faulty system.



(c) A life insurance policy pays out €120 000 if the policy holder dies and €40 000 if the policy holder becomes disabled. The insurance company has calculated that in general, in any given year, the probability of death is $0{\cdot}0001$ and the probability of disability is $0{\cdot}002$.
The company has 18 000 policy holders on its books at present who are all charged the same premium. The company's goal is to make €900 000 profit in a particular year.
Find the annual premium it should charge its customers which in an average year would generate this level of profit.

