Question 4 **(30 marks)**

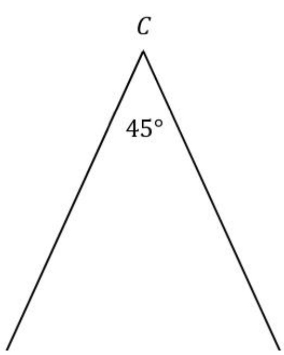
(a) **(i)** Prove that $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$.

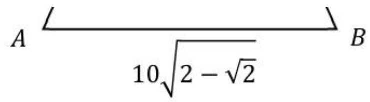
(ii) Write $\tan 15°$ in the form $\frac{\sqrt{a} - 1}{\sqrt{a} + 1}$, where $a \in \mathbb{N}$.

(b) The triangle ABC is shown in the diagram below.

$|AC| = |BC|$ and $|\angle ACB| = 45°$. $|AB| = 10\sqrt{2 - \sqrt{2}}$, as shown.

Find the length $|AC|$.




A
B
$10\sqrt{2-\sqrt{2}}$

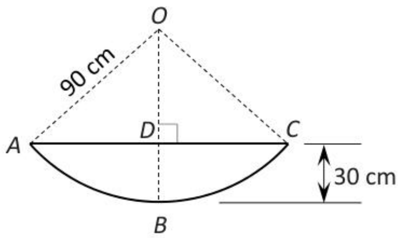


Figure 2

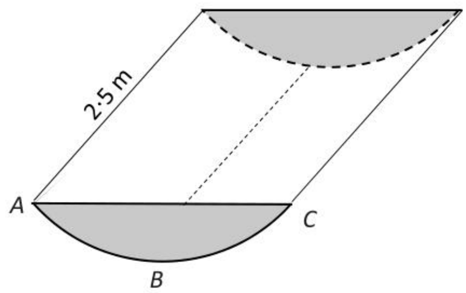


Figure 1

(i) Find $|AD|$. Give your answer in the form $a\sqrt{b}$ cm, where $a, b \in \mathbb{Z}$.

(ii) Find $|\angle DOA|$. Give your answer in radians, correct to 2 decimal places.

Question 3 **(30 marks)**

(a) Find the area of the triangle with vertices $(4, 6)$, $(-3, -1)$, and $(0, 11)$.

(b) $A(-1, k)$ and $B(5, l)$ are two points, where $k, l \in \mathbb{Q}$.

(i) Show that the midpoint of $[AB]$ is $\left(2, \frac{k+l}{2}\right)$.

(ii) The perpendicular bisector of $[AB]$ is:

$$3x + 2y - 14 = 0$$

Find the value of l and the value of k.

$l =$ ________________

$k =$ ________________

Question 5 **(30 marks)**

Rohan has a large number of small cubes. The cubes are identical in size.

(a) Some of the cubes are red, some are green, and the rest are blue.
Rohan carries out an experiment in which he picks out 5 different cubes at random.
He records the number of cubes of each colour, and replaces the 5 cubes.
He then repeats this process a number of times.

The table below shows the number of cubes of each colour the first 7 times Rohan does this, labelled Trial A to Trial G.

Trial	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Number of red cubes	0	3	2	2	4	5	1
Number of green cubes	4	2	0	3	0	0	2
Number of blue cubes	1	0	3	0	1	0	2

(i) Work out the mean and standard deviation of the number of **red** cubes per trial, for these 7 trials. Give each answer correct to 1 decimal place.

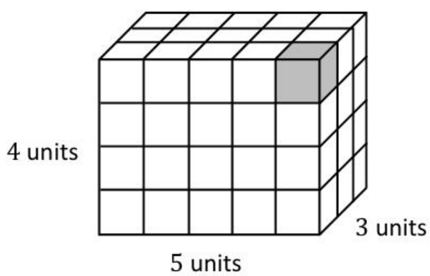
(ii) Work out the correlation coefficient between the number of **red** cubes and the number of **green** cubes per trial, for these 7 trials.
Give your answer correct to 3 decimal places.

Answer, $r =$

(iii) Rohan repeats this experiment a large number of times.
Explain why you would expect the correlation coefficient between the number of red cubes and the number of green cubes per trial to be **negative**.

(b) Rohan makes a solid cuboid of dimensions $5 \times 3 \times 4$ using his small cubes.
Each small cube has sides of length 1 unit.
Some of the small cubes have 1 face, 2 faces, or 3 faces on the outside of the cuboid.
Other small cubes have no faces on the outside of the cuboid.

In the diagram below, the small cube that is shaded has 3 faces on the outside of the cuboid.



Fill in the table below, showing the number of small cubes with 3 faces, 2 faces, 1 face, or no faces on the outside of this cuboid. Show your working out.
One of the values is filled in for you.

Number of small cubes with 3 faces on the outside of the cuboid:	
Number of small cubes with 2 faces on the outside of the cuboid:	
Number of small cubes with 1 face on the outside of the cuboid:	22
Number of small cubes with no faces on the outside of the cuboid:	

Question 1 **(30 marks)**

A group of 22 students was tested to see how far, in metres, each of them could swim without stopping for a rest. The results of the testing are shown in the ordered stem and leaf plot below. Four of the entries have been replaced with the letters a, b, c, and d.

2	b	2	7			
3	a	4	4	5	8	
4	0	1	d	5	6	9
5	2	3	7	7	8	
6	1	8	c			

Key: $2|7 = 27$ metres

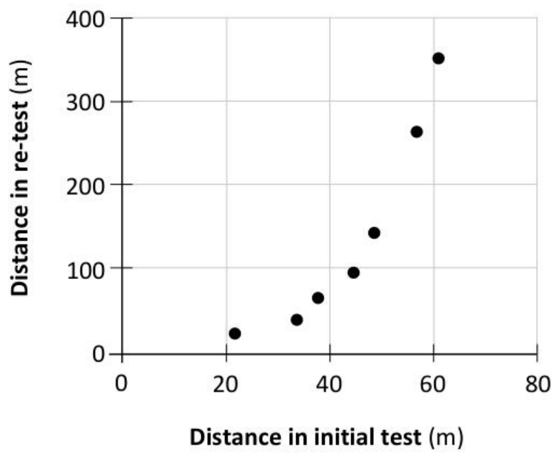
(a) **(i)** The **mode** of the data is 34 metres. Use this to write down the value of a.

(ii) The **range** of the data is 49 metres. Use this to find the value of b and the value of c.

(iii) The **median** of the data is $43{\cdot}5$ metres. Use this to find the value of d.

Seven of the 22 students took swimming lessons.
They were re-tested after the lessons, to see how far they could now swim without stopping.
The table and graph below show the results of the initial test and of the re-test for these students.

Distance in initial test (m)	Distance in re-test (m)
22	25
34	40
38	65
45	96
49	142
57	262
61	350



(b) How would you best describe how the results changed for these students, from the initial test to the re-test?

(c) The swimming coach worked out r, the correlation coefficient between the distance in the initial test and the distance in the re-test, for these seven students.
Find the value of r, correct to 4 decimal places.

$r =$

Question 3 **(30 marks)**

(a) $ABCD$ is a parallelogram.

$|AB| = 10$ cm, $|BC| = 13$ cm, and $|\angle ABC| = 110°$.

Find the area of $ABCD$, correct to the nearest cm^2.

(b) X is an angle, with $0° \leq X \leq 360°$, and

$$\cos(2X) = \frac{\sqrt{3}}{2}$$

Find **all** the possible values of X.

(c) KLM is a triangle where $|MK| = 15\sqrt{3}$ cm, $|ML| = 45$ cm, and $|\angle KLM| = 25°$.
θ is the angle $\angle LKM$.

Work out the **two** possible values of θ, for $0° < \theta < 180°$.
Give each answer correct to the nearest degree.

Question 7 **(50 marks)**

PK Hotels is a hotel chain in Europe.

(a) The ages of the people who stayed in a PK Hotel in 2023 are roughly normally distributed, with a mean age of $48 \cdot 2$ years and a standard deviation of $10 \cdot 6$ years.

(i) One person is picked at random from the people who stayed in a PK Hotel in 2023. Find the probability that this person is less than 50 years old.

(ii) Exactly 10% of people who stayed in a PK Hotel in 2023 are at least A years old. Find the value of A, correct to the nearest whole number.

(b) During their most recent stay, $\frac{1}{5}$ of PK Hotel customers used the pool.

Use this to answer parts **(b)(i)** and **(b)(ii)**.

(i) 6 of the PK Hotel customers are picked at random. Find the probability that exactly 2 of them used the pool.

(ii) n of the PK Hotel customers are picked at random, where $n \in \mathbb{N}$.

The probability that **none** of them used the pool, correct to 4 decimal places, is $0 \cdot 0047$. Work out the value of n.

(c) PK Hotels are testing a new booking system.

45% of people who log on to the PK Hotels website are shown the old booking system;

45% of people who log on to the PK Hotels website are shown the old booking system, the other 55% are shown the new booking system.
People are assigned the booking system (old or new) at random.

One third of people who see the old booking system end up booking a room.
Two fifths of people who see the new booking system end up booking a room.

One person is selected at random from those who booked a room through the PK Hotels website. Find the probability that this person used the **new** booking system.
Give your answer as a percentage, correct to the nearest percent.

This question continues on the next page

(d) In 2020, PK Hotels were rated the best hotel chain in Europe by 75% of their customers.

In 2024, PK Hotels carried out a survey of a random sample of 1000 of their customers to see if this percentage had changed. Of these, 765 rated PK Hotels the best hotel chain in Europe.

Carry out a hypothesis test at the 5% level of significance to see if this shows a change in the percentage of their customers who rate PK Hotels the best chain in Europe.

State your null hypothesis and your alternative hypothesis, state your conclusion, and give a reason for your conclusion.

Null Hypothesis:

Alternative Hypothesis:

Calculations:

Conclusion:

Conclusion:

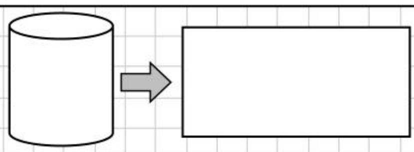
Reason for your conclusion:

Question 8 **(50 marks)**

Tommy makes ornaments from metal and glass.

(a) He makes an open metal cylinder with a height of 15 cm and a radius of 5 cm.
The **net** of this cylinder is a rectangle.

Find the dimensions of this rectangle.
Give your answers in cm, correct to 1 decimal place where appropriate.

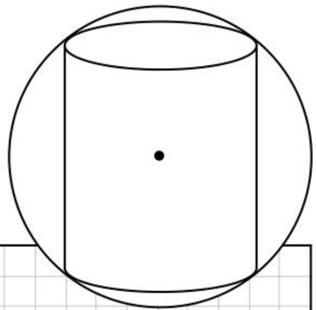


Dimensions: ____________________ by ____________________

(b) Tommy makes another cylinder with a height of 22 cm and a diameter of 12 cm.

This cylinder fits exactly inside a glass sphere.
The top and bottom edges of the cylinder touch the sphere.

Find the **volume** of the **sphere**, in cm^3, correct to 1 decimal place. Use the Theorem of Pythagoras in your solution.



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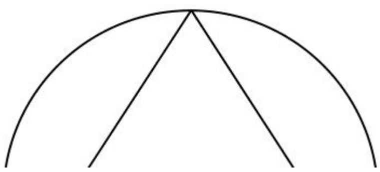
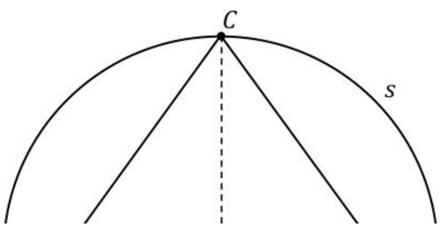
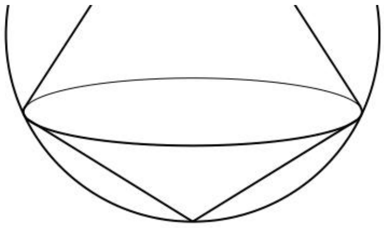
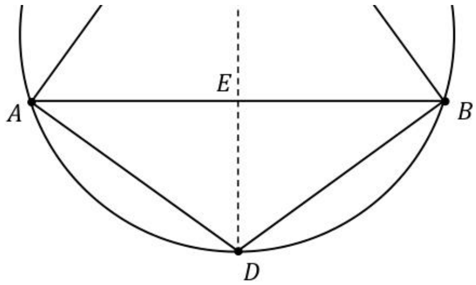
(c) Another ornament is made of two cones inscribed in a sphere.
The top cone is upright; the bottom cone is inverted. The cones have the same base.

A vertical cross-section of the ornament, taken through the centre of the sphere, shows the cones as two triangles, ABC and ADB, with a common side $[AB]$. ABC is the top cone.
The points A, B, C, and D all lie on the circle s, which represents the cross-section of the sphere. The lines AB and CD intersect at the point E.









(i) The diagram is symmetrical about the line DC. State why $|\angle CBD| = 90°$.

(ii) Hence, or otherwise, **prove** that the triangles BCE and DBE are **similar**.
Give a reason for each statement that you make, where appropriate.

There is more space for work on the next page.

(iii) The top cone has a radius of r and a height of h; that is, $|EB| = r$ and $|EC| = h$.
The sphere, represented by s, has a radius of 10 cm.

Use the similar triangles BCE and DBE to show that:

$$r^2 = 20h - h^2$$

(iv) Hence, write the volume of the top cone in terms of h and π, **and** find the value of h that gives the **maximum volume** for the top cone.

$h =$ ______________