Q1	Model Solution – 30 Marks	Marking Notes
(a)	$\frac{(4-2i)}{(2+4i)} = \frac{4-2i}{2+4i} \times \frac{2-4i}{2-4i}$ $= \frac{(8-16i-4i-8)}{2^2+4^2}$ $= \frac{-20i}{20}$ $= 0 - 1i$ $\therefore k = -1$ **OR** $\frac{(4-2i)}{(2+4i)} = \frac{-i(2+4i)}{2+4i}$ $= 0 - 1i$ $\therefore k = -1$ **OR** $4 - 2i = ki(2+4i)$ **Re**: $4 = -4k \quad \therefore k = -1$ **or Im**: $-2i = 2ki \quad \therefore k = -1$	**Scale 10C(0, 3, 7, 10)** *Note:* Accept $0 - 1i$ *Low Partial Credit:* • $2 - 4i$ • $4 - 2i = ki(2+4i)$ *High Partial Credit:* • $\frac{4-2i}{2+4i} \times \frac{2-4i}{2-4i}$ • $4 - 2i = -4k + 2ki$ • Sets Re = Re **or** Im = Im *Full Credit –1:* • $0 - i$ or $-1i$ as solution, with k not identified.

Q1	Model Solution – 30 Marks	Marking Notes
(b)	$-5 + 12i = (a+bi)^2$ $a^2 + 2abi - b^2$ Re: $a^2 - b^2 = -5$ Im: $2ab = 12 \quad \therefore \quad b = \frac{6}{a}$ $a^2 - \left(\frac{6}{a}\right)^2 = -5$ $a^4 + 5a^2 - 36 = 0$ $(a^2+9)(a^2-4) = 0$ $\therefore a = \pm 2$ and $b = \pm 3$ Answer: $2 + 3i, \ -2 - 3i$ **OR** $r = \sqrt{5^2 + 12^2} = 13$ $\tan\theta = -\frac{12}{5}$ so $\cos\theta = -\frac{5}{13}$ $(-5+12i)^{\frac{1}{2}}$ $= [13(\cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi))]^{\frac{1}{2}}$ $= \sqrt{13}\left(\cos\left(\frac{\theta}{2} + n\pi\right) + i\sin\left(\frac{\theta}{2} + n\pi\right)\right)$ $2\sin^2\left(\frac{\theta}{2}\right) = 1 - \cos\theta = 1 + \frac{5}{13}$ So $\sin\left(\frac{\theta}{2}\right) = \frac{3}{\sqrt{13}}$ and so $\cos\left(\frac{\theta}{2}\right) = \frac{2}{\sqrt{13}}$ $n = 0$: $\sqrt{13}\left(\cos\left(\frac{\theta}{2}\right) + i\sin\left(\frac{\theta}{2}\right)\right)$ $= \sqrt{13}\left(\frac{2}{\sqrt{13}} + i\frac{3}{\sqrt{13}}\right) = 2 + 3i$	**Scale 10D(0, 3, 5, 8, 10)** *Note*: Accept $2 + 3i$ for *Full Credit* *Low Partial Credit:* • $(a+bi)^2 = -5 + 12i$ • $a + bi = (-5+12i)^{\frac{1}{2}}$ • r or θ found • $-5 + 12i$ plotted on Argand diagram. • Shows some knowledge of De Moivre's theorem *Mid Partial Credit:* • Relevant equation in a single variable • Writes $-5 + 12i$ in polar form *High Partial Credit:* • Finds $a = 2$ or $b = 3$ • $-2 - 3i$ found • Correct solution in polar form (accept with mishandling of $2n\pi$)

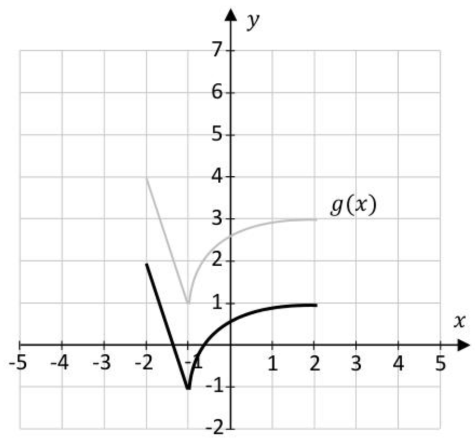
	$= \sqrt{13}\left(\frac{2}{\sqrt{13}} + i\frac{3}{\sqrt{13}}\right) = 2 + 3i$ $n = 1$: $\sqrt{13}\left(\cos\left(\frac{\theta}{2} + \pi\right) + i\sin\left(\frac{\theta}{2} + \pi\right)\right)$ $= \sqrt{13}\left(-\frac{2}{\sqrt{13}} - i\frac{3}{\sqrt{13}}\right) = -2 - 3i$	
Q1	**Model Solution – 30 Marks**	**Marking Notes**
(c)	$z^3 = r(\cos\theta + i\sin\theta)$ $z = (r(\cos\theta + i\sin\theta))^{\frac{1}{3}}$ $= 2\left(\cos\frac{\pi+2n\pi}{3} + i\sin\frac{\pi+2n\pi}{3}\right)$ $n = 0$: $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 1 + \sqrt{3}i$ $n = 1$: $z = 2\left(\cos\frac{3\pi}{3} + i\sin\frac{3\pi}{3}\right) = -2$ $n = 2$: $z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = 1 - \sqrt{3}i$	**Scale 10D (0, 3, 5, 8, 10)** *Note*: if $((r(\cos\theta + i\sin\theta))^3$ is used, award *Low Partial Credit* at most. *Note*: polar form must be used to achieve any credit *Low Partial Credit:* • $z = (r(\cos\theta + i\sin\theta))^{\frac{1}{3}}$ • r found • θ found • $-8 + 0i$ plotted on an Argand diagram • Shows some knowledge of De Moivre's theorem *Mid Partial Credit:* • $z = 8^{\frac{1}{3}}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ • $8^{\frac{1}{3}}\left(\cos\frac{\pi+2n\pi}{3} + i\sin\frac{\pi+2n\pi}{3}\right)$ *High Partial Credit:* • One root evaluated in the form $a + bi$ from De Moivre's expression • Three solutions in polar form

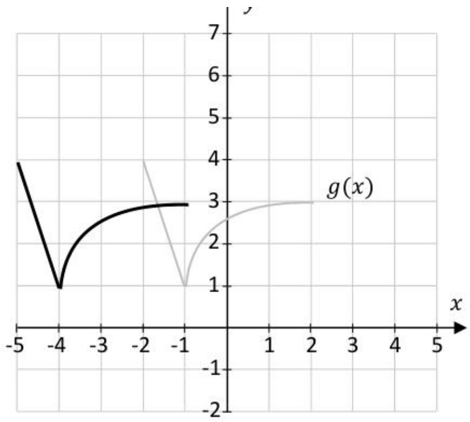
Q6	Model Solution – 30 Marks	Marking Notes
(a)	$h'(x) = a(x+1)(x-3)$ $= a(x^2 - 2x - 3)$ $(0, 6) \in h'(x)$ $\therefore 6 = a(0 + 0 - 3)$ $\Rightarrow a = -2$ $h'(x) = -2(x^2 - 2x - 3)$ $h'(x) = -2x^2 + 4x + 6$ **OR** $h'(x) = -2x^2 + 4x + 6$ y-intercept $= h'(0) = 6$ $h'(x) = -2(x+1)(x-3)$ $\therefore$ Roots = –1 and 3 **OR** $h'(x) = ax^2 + bx + c$ $h'(0) = c = 6$ So $h'(-1) = a - b + 6 = 0$ and $h'(3) = 9a + 3b + 6 = 0$ $3 \times h'(1) = 3a - 3b + 18 = 0$ So $12a + 24 = 0$ $\therefore a = -2$ and $b = 4$ i.e. $h'(x) = -2x^2 + 4x + 6$	**Scale 10D (0, 3, 5, 8, 10)** *Note*: Accept 3 points from graph verified as belonging to given equation of $h'(x)$ *Note:* three points identified from graph is *Low Partial Credit*. *Low Partial Credit:* • Identifies or uses a relevant value from the graph, for example, $(x+1)$ or 3 • A factorisation of given $h'(x)$, for example $2(-x^2 + 2x + 3)$ *Mid Partial Credit:* • Generates $(x+1)(x-3)$ from graph • Correctly verifies one point from the graph into the given $h'(x)$ • Full factorisation of given $h'(x)$ • Using simultaneous equations, finds one value (a, b, or c) *High Partial Credit:* • Generates $x^2 - 2x - 3$ from graph • From given $h'(x)$, finds two roots • Correctly verifies two points from the graph into the given $h'(x)$ • From given $h'(x)$, shows that y-intercept is 6 **and** fully factorises • Using simultaneous equations, finds two values (from a, b, and c)

Q6	Model Solution – 30 Marks	Marking Notes
(b)	$h''(x) = -4x + 4 = 0$ at max/min of $h'(x)$ $\therefore x = 1$ $h'''(x) = -4 < 0$, i.e. max $h'(1) = -2(1)^2 + 4(1) + 6 = 8$ **OR** [*Quadratic with negative x^2, so max occurs halfway between the roots:*] $x = \frac{-1+3}{2} = 1$ $h'(1) = -2(1)^2 + 4(1) + 6 = 8$ **OR** $h'(x) = -2(x^2 - 2x - 3)$ $= -2(x^2 - 2x + 1 - 1 - 3)$ $= -2((x-1)^2 - 4)$ $= -2(x-1)^2 + 8$ $\therefore$ max positive slope= 8	**Scale 10C(0, 3, 7, 10)** *Note*: It is possible to accept for *Full Credit* without $h'''(x) < 0$ *Low Partial Credit:* • Some correct differentiation of $h'(x)$ • Finds $h''(x)$ • $h'(x) = -2(x^2 - 2x - 3)$ • Indicates axis of symmetry on graph *High Partial Credit:* • $x = 1$ • $h'(x) = -2((x-1)^2 - 4)$
(c)	$h(x) = \int h'(x)\, dx$	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Accept correct answer without work.

	$h(x) = -\frac{2x^3}{3} + \frac{4x^2}{2} + 6x + C$ $(0, -2) \in h(x)$: $-2 = -0 + 0 + 0 + C$ $\Rightarrow C = -2$ $\therefore h(x) = -\frac{2x^3}{3} + 2x^2 + 6x - 2$	*Low Partial Credit:* • Any indication of integration *Mid Partial Credit:* • Integration of 3 terms fully correct *High Partial Credit:* • Relevant equation in C (with substitution)

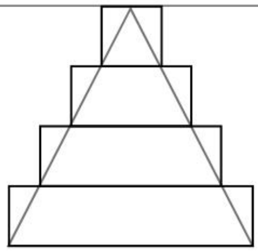
Q2	Model Solution – 30 Marks	Marking Notes
(a)	$f'(x) = 2x + b$ $f'(3) = 2(3) + b = 0$ $b = -6$ $f(3) = (3)^2 - 6(3) + c = -1$ $9 - 18 + c = -1$ $c = 8$ **OR** $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$ $-\frac{b}{2} = 3$ so $b = -6$ $-\frac{b^2}{4} + c = -1$ so $c = 8$ **OR** $f(x) = (x-3)^2 - 1$ $\quad = x^2 - 6x + 8$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • Work of merit, for example, $f(3)$ or some correct differentiation • Work of merit at completing the square • $(x-h)^2 + k$ *Mid Partial Credit:* • b correct • Uses $f(3)$ to find a correct equation in b and c • $\left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$ • Work of merit in finding both b and c • $(x-3)^2 + k$, where $k \neq -1$ • $(x-h)^2 - 1$, where $h \neq 3$ *High Partial Credit* • Finds b and work of merit in finding c • $(x-3)^2 - 1$
(b)	$\lim_{n\to\infty}\left(\frac{n}{n+1} + \frac{n+1000}{n} + \left(\frac{1}{3}\right)^n\right)$ $= \lim_{n\to\infty}\left(\frac{n}{n+1}\right) + \lim_{n\to\infty}\left(\frac{n+1000}{n}\right) + \lim_{n\to\infty}\left(\left(\frac{1}{3}\right)^n\right)$ $\left[= \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right) + \lim_{n\to\infty}\left(\frac{1+\frac{1000}{n}}{1}\right) + \lim_{n\to\infty}\left(\left(\frac{1}{3}\right)^n\right)\right]$ $= \frac{1}{1+0} + \frac{1+0}{1} + 0$ $= 2$	**Scale 5C (0, 2, 3, 5)** Note: Full credit for correct answer without work. *Low Partial Credit:* • Work of merit, for example, indicates sum of limits, divides by highest power of n in one of first two terms • Substitutes ∞ for n • Finds two or more terms of the sequence, $T_n = \frac{n}{n+1} + \frac{n+1000}{n} + \left(\frac{1}{3}\right)^n$ *High Partial Credit:* • One limit correctly evaluated and work of merit in any one of the other two limits

Q2	Model Solution – 30 Marks	Marking Notes
(c)		**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit in one part, for example, one point correctly transformed • In part (i) any vertical translation of $g(x)$ • In part (ii) any horizontal translation of $g(x)$ • In part (i) finds $g(x+2)$ or $g(x-2)$ • In part (ii) finds $g(x) - 3$ or $g(x) + 3$ *High Partial Credit:* • One part correct

		• Work of merit in both parts

Q3	Model Solution – 30 Marks	Marking Notes
(a)	Assume that $\sqrt{2}$ is rational. $\sqrt{2} = \frac{a}{b}$ where $a, b \in \mathbb{Z}, b \neq 0$ and $\text{HCF}(a, b) = 1$ $2 = \frac{a^2}{b^2}$ $2b^2 = a^2$ $\Rightarrow a^2$ is even If a^2 is even, then a is even. $\therefore a = 2k$, where $k \in \mathbb{Z}$ $2b^2 = (2k)^2$ $2b^2 = 4k^2$ $b^2 = 2k^2$ $\therefore b^2$ is even If b^2 is even, then b is even. If both a and b are even, then they have 2 as a common factor. This contradicts the assumption that $\text{HCF}(a, b) = 1$.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit, for example $\sqrt{2} = \frac{a}{b}$ • Work of merit in showing that a is even *Mid Partial Credit:* • Shows that a is even *High Partial Credit* • Shows both a and b are even
(b)	**Method 1** $\log_3 t + \frac{\log_3 t}{\log_3 9} + \frac{\log_3 t}{\log_3 27} + \frac{\log_3 t}{\log_3 81} = 10$ $\log_3 t + \frac{\log_3 t}{2} + \frac{\log_3 t}{3} + \frac{\log_3 t}{4} = 10$ $12\log_3 t + 6\log_3 t + 4\log_3 t + 3\log_3 t = 120$ $25\log_3 t = 120$ $\log_3 t = \frac{120}{25}$ $t = 3^{\frac{120}{25}} = 3^{\frac{24}{5}}$ **OR** **Method 2** $\frac{1}{\log_t 3} + \frac{1}{\log_t 9} + \frac{1}{\log_t 27} + \frac{1}{\log_t 81} = 10$ $\frac{1}{\log_t 3} + \frac{1}{2\log_t 3} + \frac{1}{3\log_t 3} + \frac{1}{4\log_t 3} = 10$ $\frac{25}{12\log_t 3} = 10$ $\log_t 3 = \frac{25}{120}$ $t^{\frac{25}{120}} = 3$ $t = 3^{\frac{120}{25}}$	**Scale 10D (0, 3, 5, 8, 10)** **3 steps:** **1.** Changing all to the same base **2.** Simplifies to an equation in $\boldsymbol{t}$ with one log **3.** Finds $\boldsymbol{t}$ *Low Partial Credit:* • Work of merit, for example, changes the base of one log (from the given equation) • Writes either 9, 27 or 81 in the form 3^k *Mid Partial Credit:* • One correct step *High Partial Credit* • 2 correct steps

Q3	Model Solution – 30 Marks	Marking Notes
(c) (i) (ii)	**(i)** *Any valid explanation, for example*: the power you need to raise 6 to, to get m. **(ii)** $\log_6 m > 1$	**Scale 10C (0, 4, 7, 10)** Note: Accept $\boldsymbol{6^x = m}$ as a valid explanation for (i) *Low Partial Credit:* • Work of merit in (i) or (ii), for example, some reference to indices • $\log_6 m > 0$ or $\log_6 m$ is positive *High Partial Credit* • (i) or (ii) correct

Q10	Model Solution – 50 Marks	Marking Notes
(a)		**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, each rectangle of height 2 units
(b)	**Method 1** $h = \frac{8}{3}$ Using similar triangles $\frac{w_1}{2} = \frac{8}{3} \div 8$ (w_1 = length of middle rectangle) $w_1 = \frac{2}{3}$ $w_2 = \frac{4}{3}$ (w_2 = length of top rectangle) Area $= 2\left(\frac{8}{3}\right) + \frac{4}{3}\left(\frac{8}{3}\right) + \frac{2}{3}\left(\frac{8}{3}\right)$ $= \frac{32}{3}$ units^2 **Method 2** 6 small triangles of length $\frac{2}{6} = \frac{1}{3}$ $\text{Area}_{\text{small } \Delta' \text{s}} = 6 \times \frac{1}{2} \times \frac{1}{3} \times \frac{8}{3} = \frac{8}{3}$ $\text{Area}_{\text{big } \Delta} = \frac{1}{2} \times 2 \times 8 = 8$ $\text{Area}_{\text{rectangles}} = \frac{8}{3} + 8 = \frac{32}{3}$ units^2 **Method 3** Sum of lengths of horizontal sides of small triangles (i.e., excess of rectangles over large triangle) $= 2$ Height of each small triangle $= \frac{8}{3}$ $\sum$ areas of small Δ's $= \frac{1}{2} \times 2 \times \frac{8}{3} = \frac{8}{3}$ Area of large triangle $= \frac{1}{2} \times 2 \times 8 = 8$ $\therefore$ Area of rectangles $= \frac{8}{3} + 8 = \frac{32}{3}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit in finding dimensions **or** area of one rectangle or one small triangle, • Finds $h = \frac{8}{3}$ • Finds the area of the large triangle *High Partial Credit:* • w_1 and w_2 found • Areas of 2 rectangles found • Sum of the areas of the small triangles found with work shown

Q10	Model Solution – 50 Marks	Marking Notes
(c)	**Method 1** $T_4 = \frac{8}{4}\left[\frac{2}{4} + \frac{4}{4} + \frac{6}{4} + \frac{8}{4}\right]$ Similarly, $T_n = \frac{8}{n}\left[\frac{2}{n} + \frac{4}{n} + \cdots + \frac{2n}{n}\right]$ $= \frac{8}{n^2}[2 + 4 + 6 + \cdots + 2n]$ $2 + 4 + \cdots + 2n$ is an A.P. with $a = 2$ and $d = 2$ and n terms $S_n = \frac{n}{2}[2(2) + (n-1)(2)]$ $= n(n+1)$ $T_n = \frac{8}{n^2}[n(n+1)]$ $= \frac{8}{n}(n+1)$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, work towards establishing pattern by writing $T_{k,}\ k \neq 3$ • $\frac{8}{n}$ mentioned as height of rectangle • Identifies $2n$ (or n) small triangles • Base length of each small triangle found *High Partial Credit:* • $T_n = \frac{8}{n}\left[\frac{2}{n} + \frac{4}{n} + \cdots + \frac{2n}{n}\right]$ or equivalent not in closed form

	or equivalent **Method 2** Total area = area of given triangle plus sum of areas of small triangles. $h = \frac{8}{n}$ $2n$ small triangles Base of each small $\Delta = \frac{2}{2n} = \frac{1}{n}$ $\text{Area of small } \Delta's = 2n \times \frac{1}{2} \times \frac{1}{n} \times \frac{8}{n} = \frac{8}{n}$ Area $= \left(\frac{8}{n}\right) + \left(\frac{1}{2} \times 2 \times 8\right)$ $= \frac{8}{n} + 8$	• Finds sum of the areas of the small triangles

Q10	Model Solution – 50 Marks	Marking Notes
(d)	$\frac{8(n-1)}{n} > 0 \cdot 95(8)$ $\frac{n-1}{n} > 0 \cdot 95$ $n - 1 > 0 \cdot 95n$ $0 \cdot 05n > 1$ $n > 20$ $n = 21$ **OR** $\frac{8n^2(n-1)}{n} > 0.95(8)n^2$ $8n^2 - 7.6n^2 - 8n > 0$ $0.4n^2 - 8n > 0$ $n^2 - 20n > 0$ $n(n-20) > 0$ $n > 20$ $n = 21$	**Scale 10D (0, 3, 5, 8, 10)** Note: where candidates multiply both sides by n^2, they must find $n = 20$ to be awarded *High Partial Credit.* *Low Partial Credit:* • Work of merit in establishing inequality, for example, finds the area of triangle *Mid Partial Credit* • Forms the correct inequality *High Partial Credit* • $n - 1 > 0 \cdot 95n$
(e) (i)	$\int_0^h \frac{x^2 c^2}{h^2} dx = \frac{c^2}{h^2} \int_0^h x^2 dx$ $= \frac{c^2}{h^2} \left[\frac{x^3}{3}\right]_0^h$ $= \frac{c^2}{h^2} \left[\frac{h^3}{3} - 0\right]$ $= \frac{c^2 h}{3}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Integral set up correctly *High Partial Credit* • Integration is correct • Mishandles $\frac{c^2}{h^2}$, but otherwise correct.

Q10	Model Solution – 50 Marks	Marking Notes
(e)(ii)	$\frac{dx}{dt} = 3 \quad \frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$ $S(x) = \frac{x^2 c^2}{h^2}$ $\frac{dS}{dx} = \frac{c^2}{h^2}(2x)$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • States a relevant derivative, for example, $\frac{dS}{dx}$ or $\frac{dS}{dt}$ • $x = \frac{h}{2}$ • Some correct differentiation

$\frac{dS}{dt} = \frac{c^2}{h^2}(2x)(3)$ $= \frac{6c^2x}{h^2}$ When $x = \frac{h}{2}$ $\frac{dS}{dt} = \frac{6c^2\left(\frac{h}{2}\right)}{h^2} = \frac{3c^2}{h}$	• Some correct differentiation *Mid Partial Credit* • Any **two** of the following: ○ $\frac{dx}{dt} = 3$ ○ $x = \frac{h}{2}$ ○ $\frac{dS}{dx} = \frac{c^2}{h^2}(2x)$ ○ $\frac{ds}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$ or similar *High Partial Credit* • $\frac{ds}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$, and any two others from the *MPC* list above

Q3	Model Solution – 30 Marks	Marking Notes
(a)	$\frac{1}{6}\sin 6x + C$	**Scale 5B (0, 2, 5)** *Partial Credit* • Some correct integration, for example, $\sin x$ *Full Credit –1* • Apply a * if the $+C$ term is missing
(b) (i)	*Co-ordinates of the point of contact* $f(2) = -21$, so point is $(2, -21)$ *Slope of the tangent at $x = 2$* $f'(x) = 6x^2 - 18x + 5$ $f'(2) = -7$ ... slope of tangent *Equation of the tangent at $x = 2$* $y - (-21) = -7(x - 2)$ or equivalent	**Scale 5D (0, 2, 3, 4, 5)** Note: Engagement with Step 3 is required to be awarded credit for Step 4 Consider solution as involving 4 steps: **1.** Finds $\boldsymbol{y}$-value at $\boldsymbol{x = 2}$ **2.** Differentiates $\boldsymbol{2x^3 - 9x^2 + 5x - 11}$ **3.** Finds $\boldsymbol{f'(2)}$ **4.** Finds the equation of the tangent *Low Partial Credit* • Work of merit, for example, some correct differentiation; Substitutes $x = 2$ in $f(x)$; Formula for the equation of a line with some relevant substitution *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q3	Model Solution – 30 Marks	Marking Notes
(b) (ii)	$f'(x) = 6x^2 - 18x + 5$ $f''(x) = 12x - 18$ $f''(x) = 0$ $12x - 18 = 0$ $x = \frac{3}{2}$ **Also accept the following for *Full Credit*:** *x values at local maximum & local minimum* $f'(x) = 0$ $6x^2 - 18x + 5 = 0$ $x_1 = \frac{18 + \sqrt{204}}{12}$ $x_2 = \frac{18 - \sqrt{204}}{12}$ *x co-ordinate of the point of inflection:* $\frac{x_1 + x_2}{2} = \frac{36}{12} \div 2$ $x = \frac{3}{2}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example, some correct differentiation of $f(x)$ or $f'(x)$; states $f''(x) = 0$ • Brings down derivative from (i) *High Partial Credit* • Correct $f''(x)$ • Finds x values of local maximum and local minimum
(c)	Slope of $l = \frac{1}{2}$	**Scale 15D (0, 4, 6, 8, 15)**

	Lines drawn parallel to l and touching the graph of $p(x)$ at $x \approx 2 \cdot 2$ and $x \approx 6 \cdot 8$	*Low Partial Credit* • Work of merit, for example, mentions slope of $l = \frac{1}{2}$ • Draws a line parallel to $l(x)$ • Draws a horizontal line at $y = \frac{1}{2}$ • Relevant work to draw graph of $p'(x)$ • Draws two parallel tangents to $p(x)$ that are **not** parallel to $l(x)$ *Mid Partial Credit* • One tangent drawn correctly *High Partial Credit* • Two tangents drawn correctly • One tangent drawn correctly and corresponding x value estimated correctly • Graphs of $l'(x)$ and $p'(x)$ shown on the diagram

Q8	Model Solution – 50 Marks	Marking Notes
(a) (i)	Top 10% means 90% below. $P(z < 1 \cdot 28) = 0{\cdot}8997$. $\frac{x-176}{36} = 1{\cdot}28$ $\Rightarrow x = 222{\cdot}08$ Minimum mark of 223	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Accept use of $P(z < 1{\cdot}29)$, to give $x = 222{\cdot}44$ *Note:* Accept answer rounded to 222 instead of 223 *Low Partial Credit:* • Mean or standard deviation indicated • z-formula with some substitution *Mid Partial Credit:* • z-score found ($1{\cdot}28$ or $1{\cdot}29$) • z-formula fully substituted $\left(\frac{x-176}{36}\right)$ *High Partial Credit:* • $\frac{x-176}{36} = 1{\cdot}28$
(a) (ii)	$P(165 < x < 210)$ $P\left(\frac{165-176}{36} < z < \frac{210-176}{36}\right)$ $= P(-0{\cdot}31 < z < 0{\cdot}94)$ [2 D.P.] $= P(z < 0{\cdot}94) - P(z > -0{\cdot}31)$ $P(z < 0{\cdot}94) = 0{\cdot}8264$ $P(z < -0{\cdot}31) = 1 - P(z < 0{\cdot}31)$ $= 1 - 0{\cdot}6217 = 0{\cdot}3783$ So answer $= 0{\cdot}8264 - 0{\cdot}3783 = 0{\cdot}4481$ $= 44{\cdot}81\%$ of 1st years got the Distinction.	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Also accept use of $-0{\cdot}30$ instead of $-0{\cdot}31$, and/or use of $0{\cdot}95$ instead of $0{\cdot}94$. *Low Partial Credit:* • Mean or standard deviation indicated • z formula with some substitution • $-0{\cdot}305$ ... or $0{\cdot}94$ ... *Mid Partial Credit:* • One relevant probability found directly from tables ($0{\cdot}6217$ or $0{\cdot}8264$) *High Partial Credit:* • $0{\cdot}3783$ found • $0{\cdot}8264$ and $0{\cdot}6217$ found *Full Credit –1:* • Uses $P(z > -0{\cdot}35)$ and finishes correctly

Q8	Model Solution – 50 Marks	Marking Notes
(b) (i)	$T = \frac{19{\cdot}8-21}{\left(\frac{5{\cdot}2}{\sqrt{60}}\right)} = -1{\cdot}787$ ...	**Scale 5C (0, 2, 3, 5)** *Note*: Accept $1{\cdot}787$ ... *Note*: $\frac{s}{\sqrt{n}}$ must be used in order to be awarded above *Low Partial Credit* *Low Partial Credit:* • Mean or standard deviation indicated • Relevant formula with some substitution *High Partial Credit:* • Formula fully substituted
(b) (ii)	p-value:	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Accept $P(z < 1{\cdot}78)$, so p-value

	$p = 2[1 - P(z < 1{\cdot}79)]$ $= 2(1 - 0{\cdot}9633)$ $= 0{\cdot}0734$ Conclusion: There is **not** enough evidence to say that the claim in the news report is incorrect [as $0{\cdot}0734 > 0{\cdot}05$]	*Note:* Accept $P(z < 1{\cdot}78)$, so p-value of $2(1 - 0{\cdot}9625) = 0{\cdot}075$. *Note:* Accept conclusion based on z-score rather than p-value. *Low Partial Credit:* • $P(z < 1{\cdot}79)$ • $0{\cdot}9633$ *Mid Partial Credit:* • $2[1 - P(z < 1{\cdot}79)]$ • Work of merit in finding p-value and correct conclusion based on this *High Partial Credit:* • p-value found but no or incorrect conclusion

Q8	Model Solution – 50 Marks	Marking Notes
(c) (i)	**Assuming no replacement:** $\frac{18}{23} \times \frac{17}{22} \times \frac{16}{21} \times \frac{5}{20}$ $= 0{\cdot}11518 \ldots = 0{\cdot}1152$ [4 D.P.] **OR** **Assuming replacement:** $\left(\frac{18}{23}\right)^3 \times \frac{5}{23}$ $= 0{\cdot}10420 \ldots = 0{\cdot}1042$ [4 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** *Note:* multiplication between relevant terms is necessary to be awarded above *Low Partial Credit* *Low Partial Credit:* • One relevant fraction *Mid Partial Credit:* • Product of four fractions, two of them correct *High Partial Credit:* • $\frac{18}{23} \times \frac{17}{22} \times \frac{16}{21} \times \frac{5}{20}$ • $\left(\frac{18}{23}\right)^3 \times \frac{5}{23}$
(c) (ii)	$\left(\frac{12}{23} \times \frac{6}{22} \times \frac{5}{21}\right) 3! = \frac{360}{1771}$ $= 0 \cdot 20327 \ldots = 0{\cdot}2033$ [4 D.P.] **OR** $\left(\frac{12}{23} \times \frac{6}{22} \times \frac{5}{21}\right) + \left(\frac{12}{23} \times \frac{5}{22} \times \frac{6}{21}\right) +$ $\left(\frac{6}{23} \times \frac{12}{22} \times \frac{5}{21}\right) + \left(\frac{6}{23} \times \frac{5}{22} \times \frac{12}{21}\right) +$ $\left(\frac{5}{23} \times \frac{6}{22} \times \frac{12}{21}\right) + \left(\frac{5}{23} \times \frac{12}{22} \times \frac{6}{21}\right)$ $= 0 \cdot 20327 \ldots = 0{\cdot}2033$ [4 D.P.] **OR** $\frac{\binom{12}{1} \times \binom{6}{1} \times \binom{5}{1}}{\binom{23}{3}} = 0{\cdot}2033$ [4 D.P.]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • One relevant fraction, for example, $\frac{12}{23}$ or $\frac{6}{22}$ or $\frac{5}{23}$ • Counts / lists different possible arrangements, for example, 3! or 6 *High Partial Credit:* • $\frac{12}{23} \times \frac{6}{22} \times \frac{5}{21}$ or any other relevant triple • Assumes keys are replaced and finishes

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$F = 3000(1+0.024)^5$ $= €3377.70$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, some correct substitution into relevant formula; finds 2.4% as a decimal
(b) (i), (ii)	**(i)** It is the amount that should be invested today to amount to €1000 in 1 years' time at the particular interest rate. **(ii)** $4000 = P(1+0.024)^6$ $\frac{4000}{1.024^6} = P$ $P = €3469 \cdot 45$	**Scale 10D (0, 3, 5, 8, 10)** Note: In (i) Accept $\boldsymbol{P} = \frac{\mathbf{1000}}{(\mathbf{1}+\boldsymbol{i})}$ *Low Partial Credit:* • Work of merit in (i) or (ii), for example, formula in (i), correct substitution into relevant formula in (ii) • 2.4% written as a decimal *Mid Partial Credit:* • (i) or (ii) correct • Work of merit in both parts *High Partial Credit* • One part correct and work of merit in the other part
(c)	$1 \cdot 024 = (1+i)^4$ $(1.024)^{\frac{1}{4}} = 1+i$ $(1.024)^{\frac{1}{4}} - 1 = i$ $0 \cdot 005947.. = i$ Rate $= 0 \cdot 59\%$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some correct substitution into relevant formula • 2.4% written as a decimal *High Partial Credit:* • $(1.024)^{\frac{1}{4}} = 1+i$ • Evaluates correctly $i = (1.024)^4 - 1$ • Uses 3 or 12 instead of 4, but otherwise correct *Full Credit -1* • Answer given as a decimal

Q8	Model Solution – 50 Marks	Marking Notes
(d) (i) (ii)	**(i)** $A(1 \cdot 0011)^{36} + A(1 \cdot 0011)^{35} + \cdots$ $\ldots + A(1 \cdot 0011)^2 + A(1 \cdot 0011)$ **OR** $= A[(1 \cdot 0011)^{36} + (1 \cdot 0011)^{35} + \cdots$ $\ldots + (1 \cdot 0011)^2 + (1 \cdot 0011)]$ **(ii)** $A[1 \cdot 0011 + (1 \cdot 0011)^2 + \cdots$ $\ldots + (1 \cdot 0011)^{35} + (1 \cdot 0011)^{36}]$ $a = 1 \cdot 0011, \quad r = 1 \cdot 0011, \quad n = 36$ $A\left[\frac{1 \cdot 0011\,(1 - 1 \cdot 0011^{36})}{1 - 1 \cdot 0011}\right] = 12000$ $A = \frac{12000}{\frac{1 \cdot 0011(1 - 1 \cdot 0011^{36})}{1 - 1 \cdot 0011}}$ $A = €326 \cdot 60$ [2 D.P.]	**Scale 15D (0, 4, 8, 12, 15)** Consider as requiring 3 steps: **1.** Finds geometric series **2.** Substitutes into geometric formula **3.** Finds A *Low Partial Credit:* • Work of merit in either part, for example, in **(i)**Writes 0.11% as a decimal; in **(ii)**, sets answer in **(i)** equal to 12000 *Mid Partial Credit:* • 1 step correct • Substantial work of merit in both parts *High Partial Credit:* • 2 steps correct *Full Credit –1:* • Correct solution, but excludes second and/or second last term

		• Investments made at the end of each month, otherwise correct
(e)	$E(x) = 11(0 \cdot 52) + (x - 5)(0 \cdot 15) + x(0 \cdot 33) = 13 \cdot 85$ $0{\cdot}15x + 0{\cdot}33x = 13{\cdot}85 - 5{\cdot}72 + 0{\cdot}75$ $0 \cdot 48x = 8 \cdot 88$ $x = €18 \cdot 50$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit, for example, some correct term in $E(x)$ *High Partial Credit* • Fully correct equation
(f)	Cost Price $=$ 82% of Selling Price Profit $=$ 18% of Selling Price Mark-up $= \frac{0{\cdot}18}{0{\cdot}82} \times 100$ $= 0 \cdot 2195 = 22\%$ [nearest percent] **OR** Let $x =$ selling price and $y =$ cost price $\frac{x - y}{x} = 0.18 \rightarrow y = 0.82x$ Mark up: $\frac{x-y}{y} = \frac{x-0.82x}{0.82x} = \frac{9}{41}$ Mark up: $\frac{9}{41} \times 100 = 21.95$ $= 22\%$ [nearest percent]	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, states CP = 82% of SP • Mentions 82% • Finds 18% of a number